String Theory and Space-Time Uncertainty Principle

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The notion of space-time uncertainty principle in string theory is clarified and further developed. The motivation and the derivation of the principle are first reviewed in a reasonably self-contained way. It is then shown that the nonperturbative (Borel summed) high-energy and high-momentum transfer behaviors of string scattering are consistent with the space-time uncertainty principle. It is also shown that, in consequence of the principle, string theories in 10 dimensions generically exhibit a characteristic length scale which is equal to the well-known 11 dimensional Planck length $g_s^{1/3}\ell_s$ of M-theory as the scale at which stringy effects take over the effects of classical supergravity, even without involving D-branes directly. The meanings of the space-time uncertainty relation in connection with D-branes and black holes are discussed and reinterpreted. Finally, we present a novel interpretation of the Schild-gauge action for strings from a viewpoint of noncommutative geometry, which conforms to the space-time uncertainty relation by manifestly exhibiting a noncommutativity of quantized string coordinates dominantly between space and time. We also discuss the consistency of the space-time uncertainty relation with S and T dualities.

§1. Introduction

Ever since string theory[1] was discovered to be the prime candidate for the unified theory including gravity[2][3], we have been opening a multitude of facets of the theory which increasingly show its richness in some unexpected ways. We are more or less convinced that the theory must have some hidden but firm foundation behind many surprising phenomena we observe on its surface. However, the present string theory still remains essentially as a collection of rules for building S-matrix in perturbation theory. We do not know why such perturbation theory can arise and what the basic principles leading to the symmetry of string perturbation theory are. Uncovering the underlying principles of string theory is an important necessary step toward the non-perturbative and completely well-defined formulations of the theory, based on which we should be able to pose various physically relevant questions that the ultimate unified theory has to answer, but hitherto have not been meaningfully dealt with.

There have been several attempts towards nonperturbative formulations of string theory. The first and most traditional one is the field theory of strings which has been pursued quite actively more than fifteen years ago[4][5][6][7]. A related approach was various attempts to the geometry of loop space and conformal field theories. A notable example is an approach based upon an abstract complex geometry[8] or, more physically, upon the renormalization group in the theory space of two-dimensional field theories[9]. All of these approaches have close connections to each other in a variety of ways depending on the directions along which we compare them. Curiously enough, however, it is generally not easy, despite their apparent similarities, to establish concrete connections among these attempts of different formulations into a unified scheme in which we can formally go back and forth among them. It seems that, from the viewpoint

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of representing string amplitudes, the difference of these approaches essentially lies in the manners that the moduli space of Riemann surfaces are decomposed[10)11)12)]. For example, the gauge invariance of string field theory expresses the requirement of smooth joining of the decomposed pieces of moduli space. The transition among different approaches therefore amounts to connecting theories with different schemes of the decomposition into one single theoretical framework. Such 'transformation theory', if successfully established, could have suggested some crucial structure behind string theory by extracting possible principles behind conformal symmetry.

A slightly different approach has been the 'old' matrix models[13)] as toy models for studying the theory on summing over the string perturbation series in lower space-time dimensions. Moreover, in recent years after the discovery of D-branes[14)] and its effective descriptions in terms of Yang-Mills theory[15)], a new approach to be called the 'new' matrix models, which can be formulated in higher space-time dimensions, has been advocated. Except for some special circumstances such as infinite-momentum limit or some sort of large N scaling limit analogous to the old matrix models, the new matrix models are regarded at best as effective low-energy descriptions of D-branes in terms of the lowest string excitation modes alone. The new matrix models have suggested some unexpected relations among local field theories embedded in string theory as low-energy approximations. The notable exceptions to this interpretation of the new matrix models might be the so-called Matrix theory[16)] and the IIB matrix model[17)]. They have been conjectured to be exact theories. However, at the present stage of development, it seems fair to say that we do not yet have succeeded in showing convincingly their validity as the fundamental formulations of string theory.

Independently of those specific attempts, one thing is evident. Namely, the structure of string theory is governed by the conformal symmetry of world-sheet dynamics, exhibited in the present perturbative rules. Indeed, almost all merits of perturbative string dynamics, such as the emergence of graviton, elimination of the ultraviolet divergences, critical dimensions, complete bootstrap between states and interactions, and so on, are direct consequences of the world-sheet conformal invariance. This is so even when we take into account various brane excitations, since the interaction of the branes are mediated by the exchange and fluctuations of strings. The motion and interaction of D-branes are described by the ordinary elementary (or 'fundamental') strings and their vertex operators in terms of the world-sheet dynamics.

The exact conformal invariance of world-sheet dynamics of strings actually corresponds to the fact that the genuine observables of string theory are solely on-shell S-matrix elements. Given only S-matrix, however, it is in general not easy to determine the real symmetries and degrees of freedom which are appropriate to express the content of the theory off shell. In particular, it is not obvious what is the appropriate generalization of the world-sheet conformal invariance to off-shell formulation of string theory. This problem is a long-standing example among the several major obstacles we encounter in trying to formulate the nonperturbative dynamics of string theory. The nature of the problem is somewhat reminiscent of the well known history of physics evolving from early quantum theory to quantum mechanics. The quantized atomic spectra were derived by imposing the Bohr-Sommerfeld quantization conditions, characterized by the adiabatic invariance, which select the particular orbits from the continuous family of allowed orbits in the phase space of classical mechanics. Then quantum mechanics replaced the Bohr-Sommerfeld condition by a deeper and universal structure, namely, Hilbert space and the algebra of observables defined on it, characterized by the superposition principle and the canonical commutation relations, respectively. We should perhaps expect something similar in string theory. Namely, the condition of conformal invariance, as the analog of the adiabatic invariance of the quantum condition, must be generalized to a more fundamental and universal structure which allows us to construct the concrete nonperturbative theory.

However, in view of the status of the past approaches to nonperturbative definition of string theory as briefly summarized above, it seems that the correct language and mathematical framework for formulating string theory remain yet to be discovered. In this situation, it is worthwhile to try to extract the most characteristic qualitative properties of string theory originated from the world-sheet conformal invariance which are universally valid irrespectively of the particular formulations of string theory. In particular, in order to seek for some clear basis behind conformal symmetry, it seems of more advantageous to express directly the smooth nature of the moduli space of Riemann surface without making decompositions of it, as string field theories, for example, are usually doing.

Although such universal qualitative properties may, by definition, be quite crude for making any quantitative predictions rather than giving rough order estimates for some simple and typical phenomena, they might be of some help for pursuing the underlying principles of string theory *if* they characterize critically the departure of string theory from the physics governed by the traditional framework of local quantum field theories. The proposal of space-time uncertainty principle[18)19)] has been motivated in this way. The purpose of the present paper is to clarify and further develop the space-time uncertainty principle of string theory from a new perspective. We first review the original derivation with some clarifications in section 2. We also make comparison of our space-time uncertainty relation with other proposals of similar nature, such as the notion of modified uncertainty relation with stringy corrections. We explain why the latter cannot be regarded as a universal relation in string theory, in contrast to our space-time uncertainty relation which will be argued here to be valid nonperturbatively. Its implications are then discussed for some aspects of string theory, mainly the high-energy limit of the string S-matrix in section 3, and then the characteristic physical scales in the physics of microscopic black holes and D-particle scatterings in section 4. The high-energy behavior with fixed scattering angle after summed over the genus by Borel sum technique is argued to almost saturate the space-time uncertainty relation. This suggests strongly that the space-time uncertainty relation is valid nonperturbatively, being obeyed independently of the strength of string coupling, at least for certain finite range of string coupling including the weak coupling regime. It is shown that the M-theory scale is a natural consequence of the space-time uncertainty relation combined with the property of microscopic black hole, without invoking D-branes. The saturation of the space-time uncertainty relation is also shown to be one of the characteristic features of D-particle scatterings. It is argued that the D-particle and anti-D-particle scattering, in general, does not saturate the space-time uncertainty relation. Section 4 contains also some remarks on the possible roles of the space-time uncertainty relation in connection with some developments of string theory, such as black-hole complementarity, holography and UV-IR correspondence. In particular, a simple interpretation of the Beckenstein-Hawking entropy for the macroscopic Schwarzschild black hole will be given from the viewpoint of the space-time uncertainty relation. Then, in section 5 we proceed to suggest a possibility of formulating the space-time uncertainty principle by quantizing string theory in a way which conforms to noncommutative geometry, exhibiting manifestly a noncommutativity between space and time. The argument is based on a novel interpretation of the string action in the Schild gauge, but its completion toward a concrete calculable scheme will be left for future works. The final section will be devoted to discussions of some issues which are not treated in the main text, including the interpretation of S and T dualities from the viewpoint of space-time uncertainty principle, and of future prospects.

In addition to developing the ideas of the space-time uncertainty relation further, it is another purpose of the present paper to discuss most of various relevant isuues in a more coherent fashion from a definite standpoint, since the previous discussions related to space-time uncertainty principle, mainly in the works by the present author and partially including some works by other authors too, were scattered in various different places. We would like to lay a foundation for further investigation by discussing their meaning and usefulness in understanding the nature of string theory. The present author also hopes that the following exposition will be useful to straighten up some confusions and awkward prejudices prevailing in the literature and to clarify the standpoint and, simultaneously, the limitations and remaining problems of the present qualitative approach. By so doing, we may hope to envisage some hints towards truly satisfactory formulations of the basic principles of string theory.

§2. Derivation of the space-time uncertainty relation

The first proposal[18)] of the space-time uncertainty relation in string theory came from an elementary space-time interpretation for the mechanism of eliminating ultraviolet divergencies in string theory. As is well known, the main reason why the string amplitudes are free from the ultraviolet divergences is that the string loop amplitudes satisfy the so-called modular invariance. The latter symmetry, which is a remnant of the conformal symmetry of Riemann surfaces after a gauge fixing, automatically generates the cutoff for the short-distance parts of the integrations over the proper times of the string propagation in loop diagrams. In traditional field theory approaches, the introduction of ultraviolet cutoff suffers, almost invariably, from the violation of unitarity and/or locality. However, string perturbation theory is perfectly consistent with (perturbative) unitarity, preserving all the important axioms for physically acceptable S-matrix, including the analyticity property of S-matrix. It should be remembered that the analyticity of S-matrix is customarily attributed to locality, in addition to unitarity, of quantum field theories. However, locality is usually not expected to be valid in theories with extended objects. From this point of view, it is not at all trivial why string theory is free from the ultraviolet difficulty and is important to give correct interpretations on its mechanism.

2.1. *A reinterpretation of energy-time uncertainty relation in terms of strings*

The approach which was proposed in[18] is to reinterpret the ordinary energy-time uncertainty relation in terms of the space-time extensions of strings,[*]

$$\Delta E \Delta t \gtrsim 1. \tag{2·1}$$

The basic reason why we have ultraviolet divergencies in local quantum field theories is that in short time region $\Delta t \to 0$, the uncertainty with respect to energy increases indefinitely $\Delta E \sim 1/\Delta t \to \infty$, which in turn induces a large uncertainty in momentum $\Delta p \sim \Delta E$. The large uncertainty in momentum means that the particles states allowed in the short distance region $\Delta x \sim 1/\Delta p$ grows indefinitely as $(\Delta E)^{D-1}$ in D-dimensional space-time. In ordinary local field theories where there is no cutoff built-in, all those states are expected to contribute to amplitudes with equal strength and consequently lead to UV infinities.

What is the difference, in string theory, with respect to this general argument? Actually, the number of the allowed states with a large energy uncertainty ΔE behaves as $\mathrm{e}^{k\ell_s \Delta E} \sim \mathrm{e}^{k\ell_s/\Delta t}$ with some positive coefficient k and $\ell_s \propto \sqrt{\alpha'}$ being the string length constant where α' is the traditional slope parameter. This increase of the degeneracy is vastly faster than that in local field theories. The crucial difference from local field theories, however, is that the dominant string states among these exponentially degenerate states are not the states with large center-of-mass momentum, but can be the massive states with higher excitation modes along strings. The excitation of higher modes along strings contribute to the large spatial extension of string states. It seems reasonable to expect that this effect completely cancels the short distance effect with respect to the center-of-mass coordinates of strings, provided that those higher modes contribute to physical processes appreciably. Since the order of the magnitude of the spatial extension corresponding to the large energy uncertainty ΔE is expected to behave as $\Delta X \sim \ell_s^2 \Delta E$, we are led to a remarkably simple relation for the order of magnitude ΔX for the fluctuation with respect to spatial fluctuation of string states participating within the time interval $\Delta T = \Delta t$,

$$\Delta X \Delta T \gtrsim \ell_s^2. \tag{2·2}$$

It is natural to call this relation 'space-time uncertainty relation'. It should be emphasized that this relation is *not* a modification of the usual uncertainty relation, but simply a *reinterpretation* in terms of strings. Note that as long as we remain in the framework of quantum mechanics, the usual Heisenberg relation $\delta x \delta p \gtrsim 1$ is also valid if it is correctly interpreted. For example, the latter is always valid for center-of-mass momentum and the center-of-mass position of strings. The space-time uncertainty relation, on the other hand, gives a new restriction on the short-distance space-time structure, which comes into play because of the intrinsic extendedness of strings. In general, therefore, we have to combine the ordinary uncertainty relation and the space-time uncertainty relations in estimating the relevant scales in string theory, as will be exhibited in later discussions.

To avoid a possible misunderstanding, we remark that, as is evident in this simple derivation, the spatial direction is dominantly the one measured along the longitudinal

[*] Throughout the present paper, our unit is $\hbar = 1, c = 1$.

direction of strings. Therefore, it should not be confused as the more familiar transverse spread of a string. That the longitudinal size indeed grows linearly with energy at least in perturbation theory is most straightforwardly seen as follows. For simplicity, let us take the case of open string. The interaction of open strings is represented by the vertex operator $\exp ipx(\tau, 0)$, inserted as the end point $\sigma = 0$ of one of the strings. If the string before the insertion is made is in the ground state with some moderate momentum, the effect of the vertex operator is to change the state after the insertion to a coherent state of the form $\exp(p\alpha_{-n}\ell_s/n)|0\rangle$ for each string mode n. This induces the contribution to the spatial extension of the string coordinate along the spatial components $\vec{p}$ of the momentum vector p_μ of order $\langle x_n^2\rangle \sim |\vec{p}|^2\ell_s^4/n^2$, which in the high-energy limit $|\vec{p}| \to \infty$ leads to $\Delta X \sim \sqrt{|\vec{p}|^2\ell_s^4 \sum_{n<n_s}(1/n^2)} \sim E\ell_s^2$. Note that here we have adopted as the measure of string extension the quantity $\sqrt{\langle \int d\sigma\, x(\sigma)^2\rangle}$. This apparently shows also that there is a large extension with respect to the time direction too. However, the interaction time ΔT should be defined with respect to the center-of-mass coordinate of strings, and hence the apparent large extension along the time direction does not directly correspond to the time uncertainty in the energy-time uncertainty relation (2·1). Furthermore, we should expect the existence of some limit $n \lesssim n_s$ for the excitation of string modes, depending on the specific region that string scattering is probing. In the Regge limit, for example, where the momentum transfer is small, we can show $\ell_s/\Delta T \sim n_s \lesssim s\ell_s^2$ (see section 3). In this case, in addition to the growth along the longitudinal direction, we have the intrinsic transverse extension of the order $\ell_s\sqrt{\sum_{n \lesssim n_s}(1/n)} \sim \ell_s\sqrt{\log(E/\ell_s)}$ for all directions corresponding to the extension of the ground state wave function. The logarithmic transverse extension is negligible compared to the linear growth. Indeed, the mechanism of suppressing the ultraviolet divergence as exhibited by the modular invariance cannot be attributed to the logarithmic growth of the extension of the ground state wave function. It is clearly the effect that is dominantly associated with the longitudinal extension of strings.

We will later see that in some cases, such as the case of high-energy-high-momentum transfer scattering of strings and D-particle scattering with slow velocities, we can effectively neglect the effect of string higher modes. This is not directly contradictory with the role of string higher modes which we emphasized above in connection with the enormous degeneracy of string states associated with the higher modes. The degeneracy refers to the standard string modes of free strings with standard boundary conditions. The situations where the string higher modes are effectively negligible occur with different backgrounds or different boundary conditions for the string coordinates as fields on string world sheet. In terms of the standard free strings, such cases are represented by a coherent state with excitation of higher string modes.

The main purpose of the present paper is to present several arguments which suggest that the space-time uncertainty relation (2·2) may be a universal principle which is valid nonperturbatively in string theory. It should be emphasized that the space-time uncertainty principle is still only qualitatively formulated. We cannot give a rigorous definition for the uncertainties ΔX and ΔT at the present stage of development. For example, one might ask how to define the time uncertainty if the string stretches linearly with energy. We always assume that the time is measured with respect to some preferred point, most naturally at the center-of-mass of a string. Also, there is no point-like probe by which we can measure the spatial uncertainty of string: String itself has an intrinsic extension depending on the scale of resolution if we are allowed to imagine a point-like probe.

The point we would like to stress is, however, that this simple looking relation has quite universal applicability both perturbatively and nonperturbatively at least in some qualitative sense, if it is interpreted appropriately. Also, involving both time and space intrinsically, the relation is not just a kinematical constraint which lessens the degrees of freedom, but in principle may place a strong constraint on the dynamics of the system. Its precise role and the correct mathematical formulation would only be found after the proper framework of string theory were established. The prime motivation for such an attitude was a general belief that any theory of quantum gravity must put some crucial restrictions at short distance scales near the Planck length beyond which the classical space-time geometry, which general relativity is based upon, is invalidated. It is then important to ascertain how such a restriction is realized in string theory, since it would suggest the precise nature how string theory is departed from the usual framework of quantum field theories. The present author is aware of many attempts in the past toward formal 'space-time quantization'. The standpoint in proposing the space-time uncertainty principle is not to propose yet another version of formal theory of quantized space-time, but to learn the possible secrets toward the unification of quantum theory and general relativity, from string theory which exhibits several ideal properties to be the unified theory in a quite surprising and unexpected fashion.

2.2. *The nature of the space-time uncertainty relation*

Now an important characteristic of the relation (2·2) is that it demands a duality between the time-like and space-like distance scales. Whenever we try to probe the short distance region $\Delta T \to 0$ in a time-like direction, the uncertainty with respect to the space-like direction increases. Not only that, we propose that the relation is also valid in the opposite limit. Namely, if we try to probe the short distance region $\Delta X \to 0$ in space-like directions, then the uncertainty ΔT in the time-like direction increases. In other words, the smallest distances probed in string theory cannot be made arbitrarily small with respect to both time-and space-like directions *simultaneously*. It was proposed in[19] that the phenomena of minimal distance[20][31] in string perturbation theory can be interpreted in this way. However, it should be kept in mind that our space-time uncertainty relation does not forbid the possibility of probing shorter distance regions than the ordinary string scale in string theory, quite contrary to the statement following the usual notion of minimal possible distance in string theory.[*] It only imposes a new condition that the short and large distances are dual to each other. We note that in some of the recent developments of nonperturbative string theory associated with D-branes, the regime of short open strings much below the string scale is a crucial ingredient.

What is the physical interpretation of the opposite limit, namely the short spatial distance which implies a large time uncertainty $\Delta T \to \infty$? Is it really possible to probe the distance scales ΔX smaller than ℓ_s? Since any string state with a definite mass has an intrinsic spatial extension of order ℓ_s, it seems at first sight impossible to do this.

[*] The possible relevance of shorter length scales has been later suggested in[21].

It turns out that the D-particle, instead of the fundamental strings, precisely plays such a role as shown later. Moreover, the fact that the asymptotic string states can be represented by vertex operators coupled with local external fields may be interpreted as a consequence of the relation $\Delta X \sim \ell_s^2/\Delta T \to 0$. In this sense, the space-time uncertainty relation can also be viewed as a natural expression of the s-t duality which has been the basic background for string theory. Roughly speaking, the resonance poles near on-shell in the s-channel corresponds $\Delta T \to \infty$, while the t-channel massless pole exchange to $\Delta X \to \infty$ with vanishingly small momentum transfer, if the exchange is interpreted in terms of pair creation and annihilation of open strings. In fact, the s-t duality was another motivation for proposing the space-time uncertainty relation in[19].

The fact that the string theory has a short distance cutoff built-in in this way might somewhat be counter intuitive, since strings have a vastly larger number of particle degrees of freedom than any local field theories or even ordinary nonlocal field theories with multi-local fields and/or some nonlocal interactions. But precisely because of this counter-intuitive nature of string theory, we have to learn the short distance structure of string theory carefully. For example, the growth of the string size along the longitudinal direction with energy might seem to be quite contrary with the familiar idea of Lorentz contraction of projectile. However, this is one of the origins why string theory contains gravity, as we will discuss in section 3. As for the large degeneracy of particle states, it should rather be interpreted that string theory suggests an entirely new way for counting the physical degrees of freedom in the region of the smallest possible distance scales. We hope that our discussion will be a basis for the concrete formulation of this general idea.

Before proceeding further, it is appropriate here to comment on the difference of our space-time uncertainty relation from the other proposal of a related uncertainty relation with stringy corrections. In parallel to the first original suggestion[*] of the space-time uncertainty relation, the high-energy behaviors of the string amplitudes have been studied. On the basis of such investigations, it was proposed independently of the proposal (2·2) that in the high-energy limit the space-time extensions of strings is proportional[22] to energy and momentum as

$$x^\mu \propto \ell_s^2\, p^\mu .$$

The reason for this proposal is that the classical solution for the string world-sheet trajectory with given wave functions with momenta p_i^μ corresponding to external asymptotic states takes the following form, in the lowest tree approximation,

$$x^\mu(z,\overline{z}) = \ell_s^2 \sum_i p_i^\mu \log|z-z_i| \tag{2·3}$$

where z_i's are the positions of the vertex operators on the Riemann surface corresponding to the asymptotic states with on-shell momenta p_i^μ. This seems also to be consistent

[*] Unfortunately, since the proposal (2·2) was initially made in a paper[18] written for the volume commemorating Prof. Nishijima's 60th birthday and has not been published in popular journals, it has long been neglected. The earliest discussion of the space-time uncertain relation in the regular journals was presented in[19]. It, however, seems that even this reference has been largely neglected to date. The author hopes that the present exposition is useful to improve the situation.

with what we have discussed using the vertex operator in our derivation of the space-time uncertainty relation. Combined with Heisenberg relation $|\delta x^\mu| \sim 1/|\delta p^\mu|$, the above proposal suggests a modified uncertainty relation[23] for each space-time component (no summation over μ)

$$|\delta x^\mu||\delta p^\mu| \gtrsim 1 + \ell_s^2 \, |\delta p^\mu|^2 \tag{2·4}$$

which leads to $|\delta x^\mu| \gtrsim \ell_s$ for *each* components of the space-time coordinates separately. Our space-time uncertaity relation (2·2) is weaker than this relation, and does not directly lead to the minimul distance, unless we assume some relation between time and spatial uncertainties : For example, if we set $\Delta T \sim \Delta X$, we immediately have the miminum distance relation $\Delta X \gtrsim \ell_s$. This is a crucial difference.

It appears that this particular form (2·4) cannot be regarded as being universally valid in string theory. We can provide at least three reasons for this. First, the uniform proportionality between energy-momenta and the extensions of the string coordinates is not valid in the region when the high-energy behavior is dominated by the Riemann surfaces where the positions of the vertex operators approach to the boundary of the moduli space. Secondly, even when the dominant contribution comes from a region which is not close to the boundary of the Riemann surface, it is known that the amplitudes after summing up the whole perturbation series using the Borel-sum technique behave differently from the tree approximation for high-energy fixed angle scattering. The known behavior is incompatible with the relation such as (2·4) demanding that the string extension grows indefinitely, while it turns out to be consistent with our relation (2·2). Thirdly and most importantly, the relation (2·4) is not effective for explaining the short-distance behaviors of D-brane interactions. In particular, the naive relation such as $|\delta x^\mu| \gtrsim \ell_s$, expressing the presence of a minimal distance, clearly contradicts the decisive role of the familiar characteristic spatial scale $g_s^{1/3}\ell_s$ in D-particle scattering, which is much smaller than the string length ℓ_s in the weak-coupling regime, and more generally in the conjecture of M-theory[24]. All of these points will be discussed fully in later sections.

One might naively think that when the spatial extension becomes large the interaction time would also increase as embodied in (2·4) , since the spatial region for interaction grows. This intuition might be correct if we were dealing with ordinary extended objects, such as polymers, which may interact each other in the bulk of the spatial extension. However, the nature of interaction of elementary strings is strongly constrained by conformal invariance. The elementary strings have no bulk-type forces among the part of strings. Thus the ordinary intuition for the extended object is not necessarily applicable to string theory. For this reason, whether the interaction time should also increase as the spatial extension or not must depend on specific situations and cannot be stated as a general property.

As the final topic of this subsection, let us ask whether and how the space-time uncertainty relation (2·2) can be compatible with kinematical Lorentz invariance. The answer is that the relation as an *inequality* can be consistent with Lorentz invariance. Suppose that the relation is satisfied in some preferred Lorentz frame which we call the proper frame where the uncertainties are $\Delta T = \Delta T_0$ and $\Delta X = \Delta X_0$ and, in particular, the spatial uncertainty can be estimated as being at rest. In most physical applications discussed later in this paper, we always assume such preferred frame in deriving the relation. Let us make a Lorentz boost of the frame of reference with velocity v and measure the same lengths in the boosted frame. Then the uncertainty in time is $\Delta T = \Delta T_0/\sqrt{1-v^2}$, while the spatial interval is contracted as $\Delta X = \sqrt{1-v^2}\Delta X_0$ or is not affected $\Delta X = \Delta X_0$ depending on the directions of the characteristic spatial scale. This shows that the inequality (2·2) is preserved in any Lorentz frame provided that it is satisfied in some proper Lorentz frame, after averaging over the spatial directions.

We can arrive at the same conclusion from a more formal consideration too. Let us temporarily suppose the existence of an algebraic realization of the space-time uncertainty relation, by introducing the space-time (hermitian) operators X^μ, transforming as Lorentz vector, as some effective agents measuring the observable distance scales in each Minkowski directions μ. Then, as has been discussed in a previous paper[25)], the space-time uncertainty relation may correspond to an operator constraint which is manifestly Lorentz invariant as given by

$$\frac{1}{2}[X^\mu, X^\nu]^2 \sim \ell_s^4 \tag{2·5}$$

where the contracted indices are summed over as usual.[*)] By decomposing into time and space components, we have

$$\sqrt{\langle -[X^0, X^i]^2\rangle} = \sqrt{\frac{1}{2}\langle -[X^i, X^j]^2\rangle + \ell_s^4} \gtrsim \ell_s^2. \tag{2·6}$$

This shows that the inequality (2·2) of the space-time uncertainty relation can in principle be consistent with Lorentz invariance, conforming to the first argument. This also suggests a possible definition of the proper frame such that the noncommutativity of space-like operators is minimized. To avoid a possible misconception, however, it should be noted that the present formal argument is *not* meant that the author is proposing that the operator constraint (2·5) is the right way for realizing the space-time uncertainty principle. In particular, it is not at all obvious whether the uncertainties can be defined using Lorentz vectors, since they are not local fields. Here it is only used for an illustrative purpose to show schematically the compatibility of the space-time uncertainty relation with Lorentz invariance. There might be better way of formulating the principle in a manifestly Lorentz invariant manner. We will later present a related discussion (section 5) from the standpoint of a noncommutative geometric quantization of strings based on the Schild action.

2.3. *Conformal symmetry and the space-time uncertainty relation*

Now we explain an independent derivation of the space-time uncertainty relation on the basis of conformal invariance of the world-sheet string dynamics, following an old work[19)] for the selfcontainedness of the present paper. This derivation seems to support our proposal that the space-time uncertainty relation should be valid universally in both short time and long time limits.

All the string amplitudes are formulated as path integrals as weighted mappings from the set of all possible Riemann surfaces to a target space-time. Therefore, any characteristic property of the string amplitudes can be understood from the property

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[*)] Similar constraints have been studied from a different viewpoint in[26)].

of this path integral. The absence of the ultraviolet divergences in string theory from this point of view is a consequence of the modular invariance. We will see that the space-time uncertainty relation (2·2) can be regarded as a natural generalization of the modular invariance for arbitrary string amplitudes in terms of the direct space-time language.

Let us start from briefly recalling how to define the distance on Riemann surface in a conformally invariant manner. For a given Riemannian metric $ds = \rho(z, \overline{z})|dz|$, an arc γ on the Riemann surface has a length $L(\gamma, \rho) = \int_\gamma \rho |dz|$. This length is however dependent on the choice of the metric function ρ. If we consider some finite region Ω and a set of arcs defined on Ω, the following definition, called 'extremal length' in mathematical literature[27)], is known to give a conformally invariant definition for a length of the set Γ of arcs,

$$\lambda_\Omega(\Gamma) = \sup_\rho \frac{L(\Gamma, \rho)^2}{A(\Omega, \rho)} \tag{2·7}$$

with

$$L(\Gamma, \rho) = \inf_{\gamma \in \Gamma} L(\gamma, \rho), \quad A(\Omega, \rho) = \int_\Omega \rho^2 dz d\overline{z}.$$

Since any Riemann surfaces corresponding to string amplitude can be decomposed into a set of quadrilaterals pasted along the boundaries (with some twisting operations, in general), it is sufficient to consider the extremal length for an arbitrary quadrilateral segment Ω. Let the two pairs of opposite sides of Ω be α, α' and β, β'. Take the Γ be the set of all connected set of arcs joining α and α'. We also define the conjugate set of arcs Γ^* be the set of arcs joining β and β'. We then have two extremal distances, $\lambda_\Omega(\Gamma)$ and $\lambda_\Omega(\Gamma^*)$. The important property of the extremal length for us is the reciprocity

$$\lambda_\Omega(\Gamma)\lambda_\Omega(\Gamma^*) = 1. \tag{2·8}$$

Note that this implies that one of the two mutually conjugate extremal lengths is larger than one.

The extremal lengths satisfy the composition law which partially justifies the naming: Suppose that Ω_1 and Ω_2 are disjoint but adjacent open regions on an arbitrary Riemann surface. Let Γ_1, Γ_2 consist of arcs in $\Omega_1 . \Omega_2$, respectively. Let Ω be the union $\Omega_1 + \Omega_2$ and Γ be a set of arcs on Ω.

1. If every $\gamma \in \Gamma$ contains a $\gamma_1 \in \Gamma_1$ and $\gamma_2 \in \Gamma_2$, then

$$\lambda_\Omega(\Gamma) \geq \lambda_{\Omega_1}(\Gamma_1) + \lambda_{\Omega_2}(\Gamma_2)$$

2. If every $\gamma_1 \in \Gamma_1$ and $\gamma_2 \in \Gamma_2$ contains a $\gamma \in \Gamma$, then

$$1/\lambda_\Omega(\Gamma) \geq 1/\lambda_{\Omega_1}(\Gamma_1) + 1/\lambda_{\Omega_2}(\Gamma_2)$$

The two cases correspond to two different types of compositions of open regions, depending whether the side where Ω_1 and Ω_2 are joined does not divide the sides which $\gamma \in \Gamma$ connects, or do divide, respectively. One consequence of the composition law is that the extremal length from a point to any finite region is infinite and the corresponding conjugate length is zero. This corresponds to the fact that the vertex operators describe the on-shell asymptotic states whose coefficients are represented by local external fields in space-time. We also recall that the moduli parameters of world-sheet

Riemann surfaces are nothing but a set of extremal lengths with some associated angle variables, associated with twisting operations, which are necessary in order to specify the joining of the boundaries of quadrilaterals.

Conformal invariance allows us to conformally map any quadrilateral to a rectangle on the Gauss plane. Let the Euclidean lengths of the sides (α, α') and (β, β') be a and b, respectively. Then, the extremal lengths are given just by the ratios

$$\lambda(\Gamma) = a/b, \quad \lambda(\Gamma^*) = b/a. \tag{2·9}$$

For a proof, see ref.[27].

Let us now consider how the extremal length is reflected on the space-time structure probed by general string amplitudes. The euclidean path-integral in the conformal gauge is essentially governed by the action $\frac{1}{\ell_s^2}\int_\Omega dz d\overline{z}\, \partial_z x^\mu \partial_{\overline{z}} x^\mu$. Take a rectangle region as above and the boundary condition ($z = \xi_1 + i\xi_2$) as

$$x^\mu(0, \xi_2) = x^\mu(a, \xi_2) = \delta^{\mu 2} B\xi_2/b,$$
$$x^\mu(\xi_1, 0) = x^\mu(\xi_1, b) = \delta^{\mu 1} A\xi_1/a.$$

The boundary condition is chosen such that the kinematical momentum constraint $\partial_1 x^\mu \cdot \partial_2 x^\mu = 0$ in the conformal gauge is satisfied for the classical solution.[*)] The path integral then contains the factor

$$\exp\Big[-\frac{1}{\ell_s^2}\Big(\frac{A^2}{\lambda(\Gamma)} + \frac{B^2}{\lambda(\Gamma^*)}\Big)\Big].$$

This indicates that the square root of extremal length can be used as the measure of the length probed by strings in space-time. The appearance of the square root is natural as suggested from the definition (2·7).

$$\Delta A = \sqrt{\langle A^2\rangle} \sim \sqrt{\lambda(\Gamma)}\ell_s, \quad \Delta B = \sqrt{\langle B^2\rangle} \sim \sqrt{\lambda(\Gamma)}\ell_s.$$

In particular, this implies that probing the short distances along both directions simultaneously is always restricted by the reciprocity property (2·8) of the extremal length, $\Delta A \Delta B \sim \ell_s^2$. In Minkowski metric, one of the directions are time-like and the other is space-like, as required by the momentum constraint. This conforms to the space-time uncertainty relation as derived in the previous subsection from a very naive argument. Also note that the present discussion clearly shows that the space-time uncertainty relation is a natural generalization of modular invariance, or of open-closed string duality, exhibited by the string loop amplitudes.

Since our derivation relies upon the conformal symmetry and is applicable to arbitrary quadrilaterals on arbitrary Riemann surfaces, which in turn can alwasy be constructed by pasting quadrilaterals appropriately , we expect that the space-time uncertainty relation is robust against possible corrections to the simple setting of our argument. In particular, the relation, being independent of the string coupling, is expected to be universally valid to all orders of string perturbation theory. Since the

*) The Hamilton constraint $(\partial_1 x)^2 = (\partial_2 x)^2$ is satisfied after integrating over the moduli parameter, which in the present case of a rectangle is the extremal length itself.

string coupling cannot be regarded as the fundamental parameter of the nonperturbative string theory, it is natural to expect that any universal principle should be formulated independently of the string coupling.

We have assumed a smooth boundary condition at the boundary of the rectangle. This led to a saturation of the inequality of the uncertainty relation. If we allow more complicated 'zigzag' shapes for boundaries, it is not possible to establish such a simple relation as above between the extremal distance and the space-time uncertainties. However, we can expect that the mean values of the space-time distances measured along the boundaries of complicated shapes in general increase, due to entropy effect, comparing with the case of smooth boundaries (namely zero mode) obtained as the average of given zigzag curves. Although there is no general proof, any reasonable definitions of the expectation value of the space-time distances conform to this expectation, since the fluctuations contribute positively to the expectation value. Thus the inequality (2·2) should be the general expression of the reciprocity relation (2·8) in a direct space-time picture. Since the relation is symmetric under the interchange $\Gamma \leftrightarrow \Gamma^*$, it is reasonable to suppose that the space-time uncertainty relation is meaningful in both limits $\Delta T \to 0$ or ∞ as we have proposed in the previous subsection.

§3. High-energy scattering of strings and the space-time uncertainties

We now proceed to study how the space-time uncertainty relation derived in the previous section is reflected in the high-energy behavior of string scattering. To the author's knowledge, careful confrontation of the space-time uncertainty relation with the high-energy (and/or high-momentum transfer) behaviors of string scattering has never been made in the past. We hope that the present section fills this gap.

3.1. *How to detect the interaction region from S-matix?*

In general, there are various difficulties in extracting precise space-time structure from on-shell S-matrix. This is so even in ordinary particle theories, since it is not possible, quantum mechanically, to define particle trajectories unambiguously only from the S-matrix element. In string theory, the difficulties are enhanced since strings themselves have intrinsic extendedness. Therefore it is not completely clear how to extract the space-time uncertainties from scattering amplitudes. Only conceivable way at present is to just treat a string state as a particle state and approximately trace its trajectory by forming a wave packet in space-time with respect to the center-of-mass coordinate. The effect of extendedness would then be approximately reflected upon the uncertainties of interaction region with respect to the center-of-mass coordinates of strings without referring to their internal structure. In our case, we have to separate the distance scales into time and spatial directions. We will see that the high-energy behavior of scattering matrix alone does not allow us to carry out it completely. But we will be able to check whether the space-time uncertainty relation is consistent or not with the high-energy behavior.

Let us consider the elastic scattering of two massless particles $1+2 \to 3+4$. The wave packet of each particle can be written as

$$\psi_i(x_i, p_i) = \int d^9\vec{k}_i \, f_i(\vec{k}_i - \vec{p}_i)\, \mathrm{e}^{i(\vec{k}_i\cdot\vec{x}_i - |\vec{k}_i| t_i)} \tag{3·10}$$

where $f_i(\vec{k})$ is any function with a peak at $\vec{k} = 0$. Here and in what follows, we assume 10 dimensional flat space-time unless otherwise specified, neglecting the issue of compactification, in particular. The inverse of the width at the peak gives the spatial extension of the wave packet. The scattering amplitude is then given as

$$\langle 3,4|S|1,2\rangle = \Big(\prod_{i=1}^{4}\int d^9\vec{k}_i\Big) f_3^*(3)f_4^*(4)f_1(1)f_2(2)\,\delta^{(10)}(k_1+k_2-k_3-k_4)A(s,t), \tag{3·11}$$

where $s=-(k_1+k_2)^2, t=-(k_2+k_3)^2$ and, for brevity of notation, the momentum variables in the wave functions $f_i(\vec{k}_i-\vec{p}_i)$ are suppressed. The uncertainty of the interaction region is measured by examining the response of S-matrix under appropriate shifts of the particle trajectories in space-time. The wave packet, after given shifts $\varDelta t$ and $\varDelta\vec{x}$ of time and spatial coordinate respectively, is

$$\psi_i(x_i,p_i;\varDelta t_i,\varDelta\vec{x}_i) = \int d^9\vec{k}_i\, f_i(\vec{k}_i-\vec{p}_i)\,\mathrm{e}^{i(\vec{k}_i\cdot\vec{x}_i-|\vec{k}_i|t_i)}\mathrm{e}^{i(\vec{k}\cdot\varDelta\vec{x}_i-|\vec{k}_i|\varDelta t_i)}. \tag{3·12}$$

To measure the uncertainty of the interaction region with respect to time, it is sufficient to choose $\varDelta t_1=\varDelta t_2=-\varDelta t_3=-\varDelta t_4=\varDelta t/2$ and $\varDelta x_i=0$ for all i. Thus, the uncertainty $\varDelta T$ can be estimated by examining the decay of the matrix element (3·11) under the insertion of the additional phase factor $\exp(-i(|\vec{k}_1|+|\vec{k}_2|)\varDelta t)$ in the integrand compared with that without the insertion. On the other hand, to measure the uncertainty of the interaction region with respect to spatial extension, it is sufficient to choose $\varDelta\vec{x}_1=-\varDelta\vec{x}_2=\varDelta\vec{x}^I/2$ and $\varDelta t_i=0$ and $\varDelta\vec{x}_3=-\varDelta\vec{x}_4=\varDelta\vec{x}^F/2$, corresponding to the relative spatial positions between the trajectories of initial and final states, respectively. In this case, the additional phase factor is $\exp[i(\vec{k}_1-\vec{k}_2)\cdot\varDelta\vec{x}^I/2-i(\vec{k}_3-\vec{k}_4)\cdot\varDelta\vec{x}^F/2)]$.

Let us choose the center-of-mass system for the momenta k_i. Assuming that the scattering takes place in the 1-2 plane and the particles are all massless, we set

$$\begin{aligned} k_1 &= (-E\sin\phi/2, E\cos\phi/2, 0,\dots,0,E),\\ k_2 &= (E\sin\phi/2, -E\cos\phi/2, 0,\dots,0,E),\\ k_3 &= (E\sin\phi/2, E\cos\phi/2, 0,\dots,0,E),\\ k_4 &= (-E\sin\phi/2, -E\cos\phi/2, 0,\dots,0,E). \end{aligned} \tag{3·13}$$

Thus,

$$k_1+k_2 = (0,0,0,\dots,0,2E), \tag{3·14}$$

$$k_1-k_2 = (-2E\sin\phi/2, 2E\cos\phi/2, 0,\dots,0,0), \tag{3·15}$$

$$k_3-k_4 = (2E\sin\phi/2, 2E\cos\phi/2, 0,\dots,0,0), \tag{3·16}$$

$$k_1-k_3 = (-2E\sin\phi/2, 0,0,\dots,0,0), \tag{3·17}$$

$$k_1-k_4 = (0, 2E\cos\phi/2, 0,\dots,0,0). \tag{3·18}$$

The order of magnitude for the decay width with respect to $|\varDelta t|$ is estimated by taking a small variation with respect to the center-of-mass energy E fixing the scattering angle ϕ, since the variation of the additional phase is just $E\varDelta t$ without containing the angle ϕ. As $\varDelta t$ increases, the decay of amplitude begins appreciably when the absolute value of the variation of logarithm of amplitude is exceeded by the variation of the additional phase $E\Delta t$. Therefore, we can roughly set

$$\Delta T \sim \langle |\Delta t| \rangle \sim \frac{1}{2}\Big|\frac{\partial}{\partial E}\log A(s,t)\Big|. \tag{3·19}$$

This way of measuring the uncertainty should perhaps be regarded as giving a lower bound, since it does not take into account the extendedness of the initial and final string states. We must evaluate this quantity at the peak values of the momenta. Note that this expression is similar to the well known Wigner formula for time delay for which we usually take only the phase of the amplitude. For the spreading of interaction region, the variation of the modulus of amplitude plays an equally important role as its phase.*)

Similarly, the decay width with respect to $|\Delta x|$ can be estimated by taking variations with respect to both the energy and scattering angle, since the additional phase now behaves as $(k_1 - k_2)\cdot \Delta\vec{x}^I/2 - (k_3 - k_4)\cdot\Delta\vec{x}^F/2 = E(-(\Delta\vec{x}^I + \Delta\vec{x}^F)_1 \sin\frac{\phi}{2} + (\Delta\vec{x}^I - \Delta\vec{x}^F)_2\cos\frac{\phi}{2})$ where the lower indices are refering the components in the 1-2 plane. The order of magnitude of the allowed spatial uncertainty is constrained by the conditions obtained by identifying the first variations of the modulus of the logarithm of the amplitude and of the additional phase

$$|\delta(E(-\Delta x_1^{(+)}\sin\frac{\phi}{2} + \Delta x_2^{(-)}\cos\frac{\phi}{2}))| = |\delta\log A| \tag{3·20}$$

for $\Delta\vec{x}^{(\pm)} \equiv \Delta\vec{x}^I \pm \Delta\vec{x}^F$. This gives two equations for determining the components of the vector $(\Delta x_1^{(+)}, \Delta x_2^{(-)}) \equiv \Delta\vec{\tilde{x}}$ from the coefficients with respect to the variations δE and $\delta\phi$. This relation shows that there are limitations in estimating the spatial uncertainties. First, since the energy variation essentially gives the same contribution to $|\Delta x^{(\pm)}|$ as Δt, the high-energy scattering of massless particles can only probe the region $\Delta X \gtrsim \Delta T$. This limitation is inevitable since, for particles moving with light velocity, a time uncertainty necessarily contributes to a spatial uncertainty of the same order. Secondly and more importantly, we can only probe the *vector* sum or difference of the spatial uncertainties for initial and final states. However, the spatial uncertainty for the space-time uncertainty relation should be defined to be the average of the uncertainties of initial and final states as $\Delta X \sim (|\Delta x^I| + |\Delta x^F|)/2$. Due to the triangle inequality, we at least have a lower bound for the spatial uncertainty

$$\Delta X > \Delta\tilde{x}. \tag{3·21}$$

Note that the equality here cannot usually be expected to be saturated, except for a very peculiar case where either of the initial or final spatial uncertainty vector vanishes. The complete information on the space-time structure could only be attained if one could completely convert the scattering matrix into the coordinate representation. Of course, for each term of the perturbation series, we already have such a picture in the form of the world-sheet path-integral representation. But nonperturbatively in general we cannot expect to have such a picture.

*) When the first variation with respect to the integration variables is small, we must be careful in checking whether the higher variations are negligible. For measuring the transverse size $|\delta x_t|$ corresponding to the shift of the form $\exp i(k_1 - k_3)\delta x_t$, the second variation is indeed important.

We remark here that the power-law behavior for high-energy fixed angle scattering necessarily leads to the decrease $1/E$ for both spatial and temporal uncertainties in the above sense. This is of course the typical behavior for the high-energy limit of local field theories.

3.2. *High-energy and high-momentum transfer behavior of string scattering*

Our task is to examine how the string scattering deviates from such typical behavior of particle scattering. Fortunately, the behaviors of string scattering in the high energy limit with fixed scattering angle have been studied in detail in ref.[22] and in ref.[28]. We study how far we can probe the short distance space-time structure using the results of these works. Throughout the present section, we use the string unit $\ell_s = 1$. At the tree level, the leading behavior is

$$A_{tree}(s,\phi) \sim ig^2 2^9 s^{-1} (\sin\phi)^{-6} \exp\Big(\frac{-s}{4} f(\phi)\Big) \tag{3·22}$$

where

$$f(\phi) = -\sin^2(\frac{\phi}{2})|\log\,\sin^2(\frac{\phi}{2})| - \cos^2(\frac{\phi}{2})|\log\,\cos^2(\frac{\phi}{2})|. \tag{3·23}$$

Although this particular form is for bosonic closed strings, the main feature that the amplitude falls off exponentially is only due to the Riemannian nature of the world sheet and hence the exponential behavior including the function (3·23) is completely universal for any perturbative string theory.

The exponential fall off of (3·22) has been regarded as one of the features of string theory which are clearly distinctive from local field theories. This has been the main motivation for the suggestion of the modification of Heisenberg uncertainty relation as (2·4). Indeed, if we apply the above method for estimating the width to the tree behavior directly with finite angle ϕ, we would get $\varDelta T \sim \varDelta X \sim E$, corresponding to the dominant classical world sheet configuration (2·3). However, the exponential fall off of tree amplitudes only means that the tree approximation is quite poor for representing the high-energy behavior of string scattering. In fact, for $N-1$ loop amplitudes, the exponential factor is replaced by $\exp(-sf(\phi)/N)$. Thus, for any small but finite string coupling, the high-energy limit is dominated by the large N contributions. The nonperturbative high-energy behavior was derived in[28] by performing the Borel-sum over N. Their final result is summarized as

$$|A_{resum}(s,\phi)| \sim \exp(-\sqrt{4sf(\phi)\log(1/g^2)}\,) \tag{3·24}$$

for $1 \ll \log(1/g^2) \ll s \ll 1/g^{4/3}$, and

$$|A_{resum}(s,\phi)| \sim \exp(-\sqrt{6\pi^2 sf(\phi)/\log s}\,) \tag{3·25}$$

for $s \gg 1/g^{4/3}$. The tree behavior (3·22) with a much faster decreasing exponential is valid only for $1 \ll s \ll \log(1/g^2)$.

Now let us estimate the space-time uncertainties exhibited in the nonperturbative high-energy behavior (3·25) for fixed string coupling. For our purpose of estimating the order of the magnitude for the decay width of the amplitudes with respect to the shift of the particle trajectories, we can neglect the imaginary part of the logarithm

$\log A(s,\phi)$, since it only contributes to the present qualitative estimation at most to the same order as the real part, and hence it only affects the numerical multiplicative factor to the width.

Using (3·19), the uncertainty of the interaction region with respect to time is

$$\Delta T \sim \sqrt{f(\phi)}. \tag{3·26}$$

We neglect the logarithms as well as the numerical factor with respect to the energy E, since our method (or any other conceivable methods) is not precise enough to include them. Note that in the limit small of scattering angle, $\Delta T \sim \phi\sqrt{\log\phi} \sim (\sqrt{t}/E)\sqrt{\log(E/\sqrt{t})} \to 0$. The dependence on the momentum transfer is strange, comparing with $\Delta T \sim 1/E$ for the standard Regge behavior for fixed t. In reality, the approximation used in the derivation of the high-energy limit will break down in the small angle limit, since in that case the saddle point approaches to the singular boundary of the moduli space. Therefore we can trust our result only for moderate scattering angles.

In order to obtain the uncertainty of the spatial interaction region, we use (3·20), which leads to

$$\epsilon_1\sqrt{f(\phi)} = -\Delta\tilde{x}_1 \sin\frac{\phi}{2} + \Delta\tilde{x}_2\cos\frac{\phi}{2} \tag{3·27}$$

$$\epsilon_2\frac{|f'(\phi)|}{\sqrt{f(\phi)}} = \Delta\tilde{x}_1\cos\frac{\phi}{2} + \Delta\tilde{x}_2\sin\frac{\phi}{2} \tag{3·28}$$

where $\epsilon_{1,2}$ are arbitrary sign factors, arising in making comparison (3·20). At $\phi = \pi/2$, the first variation with respect to the scattering angle vanishes. It is however easy to check that taking account the second variation does not change the final conclusion in the limit $E \to 0$. We then obtain

$$4(\Delta\tilde{x})^2 \sim f(\phi) + \frac{f'(\phi)^2}{f(\phi)}. \tag{3·29}$$

Because of the inequality (3·21) and (3·26), this gives a lower bound for the space-time uncertainty relation as

$$\Delta T\Delta X > \Delta T\Delta\tilde{x} \sim \frac{1}{2}\sqrt{f(\phi)^2 + f'(\phi)^2} = \sqrt{\sin^2\frac{\phi}{2}(\log\sin\frac{\phi}{2})^2 + \cos^2\frac{\phi}{2}(\log\cos\frac{\phi}{2})^2}. \tag{3·30}$$

For moderate values of scattering angle which is not close to 0 or π, the right hand side is of order one, independent of energy. This is consistent with our space-time uncertainty relation. In particular, this shows that we cannot probe arbitrarily short distances even if both energy and momentum transfer increase without limit. The fact that the right hand side vanishes in the limit $\phi \to 0$ or π implies only that this inequality (3·21) is far from being saturated. For example, if we use (3·27) and (3·28) in the limit $\phi \to 0$ of forward scattering, we find $\Delta\tilde{x}_2 \to \sin\frac{\phi}{2} \to 0$ and $\Delta\tilde{x}_1 = O(1)$ which indicate that the components of the spatial uncertainty match between the initial and final states, $\Delta x_2^I \sim \Delta x_2^F$, along the longitudinal direction while along the transverse direction there is a spread of order one. In any case, however, we cannot trust our formulas for such small scattering angle, as emphasized already. For generic scattering angle, it seems reasonable to regard the inequality as almost saturated, since there is no preferred directions for the spatial uncertainty.

3.3. *The Regge limit and space-time uncertainties*

We have studied the high-energy limit for fixed scattering angle, which means high-momentum transfer $s \sim t \to \infty$. Let us briefly consider the case of fixed momentum transfer $t = -(k_1 - k_3)^2 = -4E^2 \sin^2 \frac{\phi}{2}$, namely a limit of small scattering angle.*) Since this corresponds to extreme limit of small scattering angle, the above discussion suggests that we cannot expect information more than some matching conditions between initial and final spatial uncertainties. The high energy behavior is dominated by the exchange of Regge poles. In string theory, the leading Regge trajectory is that of graviton. Hence, the tree (invariant) amplitude is given by

$$A_{tree}(s,t) \sim g^2 \frac{1}{t}(-is/8)^{2+t/4}. \tag{3·31}$$

As is well known, however, this behavior is actually incompatible[30] with unitarity for sufficiently high energies. To recover unitarity, it is again necessary to resum the whole perturbation series. This problem has been investigated in ref.[31] using the method of Reggeon calculus. It was shown that the series can be summed into an (operatorial) eikonal form in the region of large impact parameter, or equivalently, in the region of small momentum transfer in the present momentum representation. In particular, the tree form (3·31) is justified only when the eikonal is very small where $1/b \sim \sqrt{t} < (g^2 s)^{-1/(D-4)} \ll 1$ is satisfied. This is essentially the classical region. By applying the same method as above to the tree amplitude (3·31) in this region, we obtain the uncertainty in time,

$$\Delta T \sim \frac{\partial}{\partial E} \log s \sim 1/E \ll 1. \tag{3·32}$$

For the uncertainty in spatial direction, we can only obtain the following constraints,

$$|\Delta\tilde{x}_1| \sim \frac{2}{E}\Big((1+\frac{t}{2}\log E)\sin\frac{\phi}{2} + \frac{\epsilon}{\sin\frac{\phi}{2}}(1-\frac{t}{2}\log E)\Big) \sim \frac{1}{\sqrt{t}}(1-\frac{t}{2}\log E), \tag{3·33}$$

$$|\Delta\tilde{x}_2| \sim \frac{2}{E}\Big((1+\frac{t}{2}\log E)\sin\frac{\phi}{2} - \frac{\epsilon}{\sin\frac{\phi}{2}}(1-\frac{t}{2}\log E)\Big) \sim \frac{4}{E} \ \mathit{or} \ \frac{-2t}{E}\log E, \tag{3·34}$$

where ϵ is the choice of relative sign between the energy and angle variations in making comparison of (3·20). In conformity with the tendency found in the fixed-angle case, the spatial uncertainties along the longitudinal direction 2 match between initial and final states. This is as it should be since the space-time uncertainty relation requires that the longitudinal spatial uncertainty would increase with energy (or decrease of interaction time). The growth of the longitudinal size of string with decreasing time uncertainty would be impossible unless the uncertainties match between the initial and final states along that direction. On the other hand, along the transverse directions,

*) For a previous analysis of high-energy string scattering with fixed momentum transfer, see[29].

(3·33) indicates that the average uncertainty spreads without limit as the momentum transfer vanishes, corresponding to the exchange of massless graviton. The singular behavior of (3·33) in t is produced by the pole at $t = 0$ of the Regge amplitude. This is also consistent with the growth of the longitudinal length of strings. From the s-channel viewpoint, it is very difficult to imagine the generation of long-range interactions without the rapid growth of the string extension.

That the high-energy Regge behavior of string amplitudes, at least only with its simplest 2-2 elastic scattering, can only be utilized for a consistency check of the space-time uncertainty relation might seem somewhat disappointing. However, this is inevitable in view of the number of the variables available in the scattering amplitude and its kinematical structure.

We note that our method of estimating the interaction region directly from the high-energy behavior is not sensitive enough to fix the Regge intercept: For the property $\Delta T \sim 1/E$, only the power behavior with respect to energy with fixed momentum transfer is relevant and the value of intercept, including its sign, cannot be detected. Actually in the Regge limit, this information of the Regge intercept, namely, the existence of massless states such as graviton and photon in string theory, may be regarded as a consequence of the space-time uncertainty relation. It has long been known[32)] in light-cone string theory that there is a simple geometrical explanation for the intercept of the Regge trajectory of string theory. We can adapt this geometrical interpretation for the space-time uncertainty relation as follows.

Consider the elastic scattering of two strings, $p_1 + p_2 \to p_3 + p_4$ in the extreme forward region where the longitudinal momenta p_1^+ and p_3^+ is very large compared with others. Namely, we choose a sort of the laboratory frame instead of the center-of-mass frame. If we treat the high-momentum state as the target string and the low-momentum state as the projectile string, it is natural to represent the interaction by the insertion of the vertex operators corresponding to the absorption and emission of the projectile string onto the target string state. In this case, the projectile string can effectively be treated as a probe with small longitudinal extension, since the momentum associated with the vertex operator is small. On the other hand, by reversing the roles of projectile and target strings we see that the intermediate state induced by the interaction has a large longitudinal extension. Also, by repeating the foregoing analysis of the Regge limit in the present frame, we can see that the interaction time is small and the longitudinal extension associated with initial and final states must match to each other in the Regge limit. Note that main difference of the situation from the center-of-mass frame is only $s \sim p_1^+ p_2^-$, instead of $s \sim E^2$. This means that the probability for the interaction for the forward scattering is proportional to its longitudinal length which can be regarded as being proportional to the longitudinal momenta, since the interaction of strings are regarded as occurring independently at each segment of the target string.[*)] With the identification of the longitudinal length and the longitudinal momentum in accordance

[*)] Here it is important that the string is a continuous object. If, for example, we consider some object with only discrete and finite number degrees of freedom, such as the old bilocal field theory, we cannot expect to generate graviton or any massless particles naturally even if the nonlocality is managed to increase as the interaction time decreases. As this remark suggests, it seems very difficult to construct reasonable theoretical framework other than string theory such that it contains gravity and satisfies the space-time uncertainty relation.

with the space-time uncertainty relation, this means that the probability amplitude (or the total cross section using optical theorem) in the tree approximation linearly grows with the large longitudinal momentum $p_1^+ \sim p_3^+$. For the invariant amplitude this amounts to the Regge intercept $\alpha(0) = 2$. If we only consider the open string interactions neglecting the closed string, the same argument leads to $\alpha(0) = 1$, since the interaction only occurs at the string end point and hence the probability is constant in the high-energy limit.

3.4. *A remark: minimum nonlocality ?*

Finally we remark that there is no guarantee that the Borel summation of the leading behaviors of the perturbation series gives the unique nonperturbative answer. Therefore, the formula which we have relied upon for studying the fixed angle scattering may not be completely correct due to some nonperturbative effects that have not been taken into account in the Borel summation.

However, at least for a certain finite range of string coupling including the weak coupling regime, it seems reasonable to regard the properties found here as evidencing the following viewpoint on the space-time uncertainty relation. Namely, the space-time uncertainty relation is a natural principle which characterizes string theory nonperturbatively as being *minimally but critically* departed from the usual framework of local field theory for resolving the ultraviolet difficulties. This view may be supported by recalling that the high-energy behavior (3·25) with fixed scattering angle almost saturates the fastest allowed decrease of the form, $\exp(-f(\phi)\sqrt{s}\log s)$, derived in ref.[33]. The proof of this theorem uses, apart from the usual analyticity and unitarity, the assumptions of polynomial boundedness in the energy for fixed t and also of the existence of a mass gap. The latter is not certainly satisfied in the presence of graviton. However, that this lower bound is just the behavior, up to logarithms, corresponding to a saturation of the space-time uncertainty relation, as is exhibited by (3·30) is very suggestive. We may say that 'locality' is almost satisfied in some effective sense in string theory from the viewpoint of analyticity property of scattering amplitudes.[*] The space-time uncertainty relation may be interpreted as the basic principle for introducing nonlocality in a way which does not contradict the property of scattering amplitude whose validity is usually expected for local field theories.

§4. The meaning of space-time uncertainty relation

Now that we have checked the consistency of the space-time uncertainty relation with high-energy string scattering, let us study its implications from a more general standpoint. Since the relation expresses a particular way by which string theory deals with the short distance structure of space-time, we expect that it should predict (or explain) some characteristic features of string theory, when combined with other physical characteristics of the theory.

[*] In the literature, we can find another approach to locality in string theory based on the commutation relation of string fields[34][35]. It would be an interesting problem to connect the latter approach to our space-time uncertainty relation.

4.1. *Characteristic scale for microscopic black holes in string theory*

We first consider an implication to microscopic gravitational phenomenon. Usually, the characteristic scale of quantum gravity is assumed to be the Planck scale, which in ten dimensional string theory is equal to $\ell_P \sim g_s^{1/4}\ell_s$ corresponding to the ten dimensional Newton constant $G_{10} \sim g_s^2\ell_s^8$. Indeed, if we neglect the effect of higher massive modes of string theory, this would be the only relevant scale. Let us consider what is the limitation for the notion of classical space-time from this viewpoint against the possible formation of black holes in the short distance regime. Suppose that we probe the space-time structure at a small resolution of order δT along the time direction. This necessarily induces an uncertainty $\delta E \sim 1/\delta T$ of energy. Let us require that the ordinary flat space-time structure is qualitatively preserved at the microscopic level by demanding that no virtual horizon is encountered, associated with this uncertainty of energy. Then we have to impose the condition that the minimum resolution along spatial directions must be larger than the Schwarzschild radius corresponding to this energy.

$$\delta X \gtrsim (G_{10}/\delta T)^{1/7},$$

leading to a 'black-hole uncertainty relation' *) ,

$$\delta T(\delta X)^7 \gtrsim G_{10}. \tag{4·35}$$

This expresses a limitation, for observers at asymptotic infinity, with respect to spatial and temporal resolutions below which the naive classical space-time picture without the formation of microscopic black holes can no longer be applied. If we assume that the spatial and temporal scales are of the same order, this would lead to the familiar looking relation $\delta T \sim \delta X \gtrsim \ell_P$. However, in the presence of some stable massive particle state in probing the short distance scales such as D-particle, this assumption may not be necessarily true, and we should in general treat the two scales independently.

Furthermore, it is important to remember that the relation (4·35) does *not* forbid *smaller* spatial scales than δX *in principle*. Suppose we use as a probe a sufficiently heavy particle, such as a D-particle in the weak string-coupling regime. We can then neglect the extendedness of wave function and localize the particle in an arbitrarily small region. In this limit, classical general relativity can be a good approximation. But general relativity only requires the existence of local time, and hence we cannot forbid the formation of black holes. It only says that we cannot read the clock on the particle inside the black hole from asymptotic region at infinity. If we suppose a local observer (namely just another particle) sitting somewhere apart in a local frame which falls into the black hole, it is still meaningful to talk about the local space-time structure at scales which exceed the condition (4·35), since the extendedness of wave packet of a sufficiently heavy particle can, in principle, be less than the limitation set by (4·35). In connection with this, it should be kept in mind that the above condition only corresponds to the restriction for the formation of microscopic black holes. For example, for light probes instead of a very heavy one, we have to take into account the usual quantum mechanical spread of wave functions, as we will do later in deriving the characteristic scale of D-particle scattering.

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*) Similar relations have been considered by other authors independently of string theory. However our interpretation is somewhat different from other works. See for example[36].

Despite a similarity on its appearance to (4·35), the space-time uncertainty relation of full string theory places the limitation in principle, beyond which we can never probe the space-time structure by any experiments allowed in string theory;

$$\Delta T \Delta X \gtrsim \ell_s^2. \tag{4·36}$$

Note that such a strong statement is acceptable in string theory, because it is a well-defined theory resolving the ultraviolet problems. The nature of the condition (4·35) is therefore quite different from (4·36). In this situation, the most important scale corresponding to the truly stringy phenomena is where these two limitations of different kinds meet. Namely, beyond this crossover point, it becomes completely meaningless to talk about the classical geometry of black hole, and hence it is where the true limitation on the validity of classical general relativity must be set. The critical scales ΔT_c and ΔX_c corresponding to the crossover are obtained by substituting the relation $\Delta T_c \sim \ell_s^2/\Delta X_c$ into (4·35) :

$$(\Delta X_c)^6 \sim \frac{G_{10}}{\ell_s^2} = g_s^2 \ell_s^6 \tag{4·37}$$

which gives

$$\Delta X_c \sim g_s^{1/3}\ell_s, \quad \Delta T_c \sim g_s^{-1/3}\ell_s. \tag{4·38}$$

Interestingly enough, we have derived the well known eleven dimensional M-theory scale

$$\ell_M = g_s^{1/3}\ell_s = \Delta X_c \tag{4·39}$$

as the critical spatial scale, without invoking D-branes and string dualities directly.

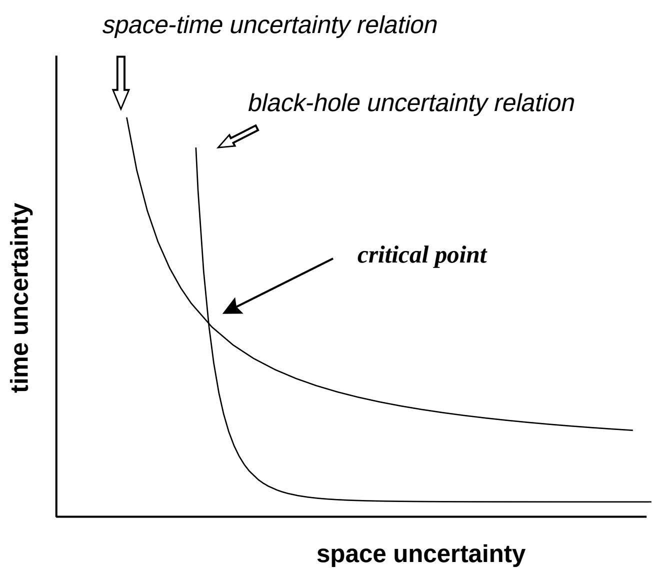

Fig. 1. This diagram schematically shows the structure of the space-time uncertainty relation and the black hole uncertainty relation. The critical point is where the two relations meet.

To appreciate the meaning of the critical scales, it is useful to look at the diagram in Fig. 1. We see clearly that for $\Delta t < \Delta T_c$ there is no region where the concept of microscopic black hole associated with quantum fluctuation is meaningful. On the other hand, in the region $\Delta t > \Delta T_c$, there is a small region where $(\Delta t)^{-1}\ell_s^2 < \Delta x < \Delta X_c$ is satisfied and hence the black hole formation at the microscopic level may be meaningful in string theory. The importance of this region increases as the string coupling grows larger. In the limit of weak string coupling where $\Delta T_c \to \infty$ and $\Delta X_c \to 0$, there is essentially no fluctuations of space-time metric corresponding to the formation of microscopic black holes. Unfortunately, the space-time uncertainty relation alone cannot predict more detailed properties of stringy black holes at microscopic scales. It is an important future problem to explore the physics in this region in string theory.

4.2. *Characteristic scale of D-particle dynamics*

In the case of high-energy string scattering, we could not probe the region $\Delta X < \Delta T$. To overcome this barrier, we need massive stable particles. The point-like D-brane, *i.e* D-particle, of the type IIA superstring theory is an ideal agent in this context, at least for sufficiently weak string coupling, since its mass is proportional to $1/g_s$ and its stability is guaranteed by the BPS property. The derivation of the characteristic scale of D-particle interactions has been given in two previous works[37)38)]. However, for the purposes of selfcontainedness and for comparison with the result of the previous subsection, we repeat the argument here with some clarifications.

Suppose that the region we are trying to probe by the scattering of two D-particles is of order ΔX. Since the characteristic spatial extension of open strings mediating the D-particles is then of the order ΔX, we can use the space-time uncertainty relation. The space-time uncertainty relation demands that the characteristic velocity v of D-particles is constrained by

$$\Delta T \Delta X \sim \frac{(\Delta X)^2}{v} \gtrsim \ell_s^2,$$

since the period of time required for the experiment is of order $\Delta T \sim \Delta X/v$. Note the last relation is due to the fact that ΔT is the time interval during which the length of the open string is of the order ΔX. This gives the order of the magnitude for the minimum possible distances probed by the D-particle scatterings with velocity v.

$$\Delta X \gtrsim \sqrt{v}\ell_s. \tag{4·40}$$

Thus to probe shorter spatial distances than the time scale, $\Delta X \ll \Delta T$, we have to use D-particles with small velocity $v \ll 1$. However, the slower the velocity is and hence the longer the interaction time is, the larger is the spreading of the wave packet.

$$\Delta X_w \sim \Delta T \Delta_w v \sim \frac{g_s}{v}\ell_s \tag{4·41}$$

since the ordinary time-energy uncertainty relation says that the uncertainly of the velocity is of order $\Delta_w v \sim g_s v^{-1/2}$ for the time interval of order $\Delta T \sim v^{-1/2}\ell_s$. To probe the range of spatial distance ΔX, we must have $\Delta X \gtrsim \Delta X_w$. Combining these two conditions, we see that the shortest spatial length is given by

$$\Delta X \sim g_s^{1/3}\ell_s \tag{4·42}$$

and the associated time scale is

$$\Delta T \sim g_s^{-1/3}\ell_s. \tag{4·43}$$

Thus the minimal scales of D-particle-D-particle scattering coincide with the critical scales for microscopic black holes derived above. In other words, the minimal scales of D-particle scattering is just characterized by the condition that the fluctuation of the metric induced by the D-particle scattering is automatically restricted so that no microscopic black holes are formed during a scattering process. Indeed, we can get the same scales from the black-hole uncertainty relation (4·35) by putting the restriction $\delta T/m\delta X \sim \delta X$ for the spreading of the wave packet of a free particle with mass $m \sim 1/g_s\ell_s$ which is localized in the spatial uncertainty of order δX, conforming to the above agreement.

In view of this interpretation of the scale of D-particle dynamics, the agreement between D-particle scales and those for microscopic black hole formation is consistent with a seemingly surprising fact that the effective supersymmetric Yang-Mills quantum mechanics, which is formulated on a flat Minkowski background and does not at least manifestly have any degrees of freedom of gravitational field, can reproduce[39)40)41)] the gravitational interaction of type IIA supergravity, or equivalently, of the 11 dimensional supergravity with vanishingly small compactification radius $R_{11} = g_s\ell_s$, in the weak string-coupling (perturbative) regime. Naively, we expect that the supergravity approximation to string theory is only valid at scales which are larger than the string scale ℓ_s. On the other hand, the Yang-Mills approximation keeping only the lowest string modes is in general regarded as being reliable in the regime where the lengths of open strings connecting D-particles are small compared with the string scale. However, the consideration of the previous subsection indicates that truly stringy gravitational phenomena are characterized by the critical scales $\Delta T_s \sim g_s^{-1/3}\ell_s \gg \ell_s, \Delta X_c \sim g_s^{1/3}\ell_s \ll \ell_s$. Given the fact that the Yang-Mills approximation to string theory is characterized precisely by the same scales, the compatibility of Yang-Mills approximation with supergravity can naturally be accepted as a consistency check of our chain of ideas at a 'phenomenological' level.

It should be kept in mind that the present discussion is not of course sufficient for explaining the agreement of the Yang-Mills description with supergravity at long distance regime. Why such Yang-Mills models can simulate gravity is still largely in the realm of mystery, since Yang-Mills theory has no symmetry corresponding to general coordinate transformation and also that it has no manifest Lorentz invariance, either as an effective 10D theory or as an infinite-momentum frame description of 11D theory following the Matrix-theory conjecture. At least in the lowest order one-loop approximation[39)], the agreement is explained by the constraint coming from supersymmetry. It seems hard to believe, however, that the global supersymmetry *alone* can explain the quantitative agreement of 3-body interactions found in[41)] which is a genuinely non-linear effect of supergravity. But this might turn out to be a wrong prejudice. For a recent detailed discussion on the role of supersymmetry in general Yang-Mills matrix models, see[42)] and references cited therein.

As a next topic of this subsection, we consider the D-particle scales from a slightly different viewpoint of degrees-of-freedom counting. Although the space-time uncertainty relation is first derived by a reinterpretation of the ordinary quantum mechanical uncertainty relation between energy and time, the fact that it puts a limitation on the observable length scales suggests that it should also imply a drastic modification on the counting of physical degrees of freedom. Let us consider how it affects the quantum state itself in the case of D-particles. The discussion of the previous subsection on D-particle scales emphasized the possible scale probed by a dynamical process of scattering. It is not obvious whether the same scale is relevant for restricting the general quantum state. The following derivation of the scale suggests that the same scale indeed is important from this viewpoint too.

Consider a state of a D-particle which is localized within a spatial uncertainty ΔX. The ordinary Heisenberg relation $\Delta p \Delta X \gtrsim 1$, which is the usual restriction on the degree of freedom in quantum theory, then gives the order of magnitude for the velocity

$$v \gtrsim \frac{g_s \ell_s}{\Delta X}.$$

On the other hand, the space-time uncertain relation reflecting the interaction of D-particles through open strings implies the condition (4·40) for the minimum meaningful distances among D-particles with given velocities of order v as

$$\Delta X \gtrsim \sqrt{v} \ell_s.$$

Thus again we have the same restriction on the scale of localization $\Delta X \gtrsim g_s^{1/3} \ell_s$ of a D-particle for a restriction to quantum state too. In the M-theory interpretation of D-particle, this is consistent with the holographic behavior that the minimum bit of quantum information stored in a D-particle state is identified with the unit of cell whose volume in the transverse dimensions is of order of 11 dimensional Planck volume $\ell_{11}^9 \sim g_s^3 \ell_s^9 \sim (\Delta X)^9$.

Finally, we explain how these characteristic scales of D-particle dynamics are embodied in the effective Yang-Mills quantum mechanics: It can best be formulated by a symmetry property, called ‘generalized conformal symmetry’ which was proposed in the work[43] and further developed in[44] and[45][46]. Briefly, the effective action, suppressing the fermionic part,

$$S = \int dt \, \mathrm{Tr}\Big(\frac{1}{2 g_s \ell_s} D_t X_i D_t X_i + i\theta D_t \theta + \frac{1}{4 g_s \ell_s^5} [X_i, X_j]^2 - ....) \tag{4·44}$$

of the supersymmetric Yang-Mills matrix quantum mechanics is invariant under the transformations

$$X_i \to \lambda X_i, \ t \to \lambda^{-1} t, \ g_s \to \lambda^3 g_s, \tag{4·45}$$

$$\delta_K X_i = 2\epsilon t X_i, \delta_K t = -\epsilon t^2, \ \delta_K g_s = 6\epsilon t g_s, \tag{4·46}$$

which together with the trivial time translation symmetry form an SO(2,1) algebra. This shows that the characteristic scales of the theory are indeed nothing but (4·42) and (4·43). Combining with the fact that the same symmetry is satisfied in the classical metric of the D0 solution of type IIA supergravity and with a help of some constraints due to supersymmetry, it was demonstrated in[43] that the generalized conformal symmetry can determine the effective D0 action as a probe to all orders in velocity expansion, within the eikonal approximation neglecting time derivatives of velocity.

An important remark here is that the supersymmetry of the model plays a crucial role for ensuring that the D-particles can be free when the distances among them are sufficiently large. Without the supersymmetry we would have nonvanishing zero-point energies. The zero-point energies contribute to the effective static potential which grows linearly as distances. That would render the scattering experiment impossible. If we assume the scaling symmetry (4·45), the effective for two-body scattering in general takes the following form in the limit of weak coupling

$$S_{eff} = \int dt\Big(\frac{1}{2g_s\ell_s}v^2 - \sum_{p=0}^{\infty} c_p \frac{v^{2p}\ell_s^{4p-2}}{r^{4p-1}} + O(g_s)\Big) \tag{4·47}$$

with c_p being numerical constants. The zero-point oscillation corresponds to the first term $p = 0$. It is well known that the supersymmetry eliminates the next term $p = 1$ too, and the effective interaction starts from the $p = 2$ term v^4/r^7.

As a further remark, we note that the product $\delta X\delta t$ of small variations is invariant under the above transformations, suggesting that the generalized conformal symmetry may be a part of more general transformations which characterize the algebraic structure associated with the space-time uncertainty relations. Just like the canonical structure of classical phase space is transformed into Hilbert space of physical states in quantum theory which is characterized by the 'unitary structure', such a characterization might lead to some appropriate mathematical structure underlying the space-time uncertainty relation. Exploring along this direction might be an important future direction. However, this issue will not be addressed in the present paper. To carry out it meaningfully, we definitely need more data.

For example, the Yang-Mills models of above type cannot describe the system with both D-branes and anti D-branes. Once the D-branes and anti D-branes are both included[47)], we have no justification for the approximation retaining only the lowest string modes, as the following argument shows. In the simplest approximation of retaining only the usual gravitational interaction, the effective action is

$$\int dt\Big(\frac{1}{2g_s\ell_s}v^2 + \frac{\ell_s^6}{r^7}\Big)$$

If we assume that the space-time uncertainty relation is saturated, the relation $r^2 \sim v\ell_s^2$ leads to an estimate of the characteristic length scale as $r_c \sim g_s^{1/11}\ell_s$ which is smaller than the string scale ℓ_s in the weak coupling region, while it is somewhat larger than the critical spatial scale of D-particle-D-particle scattering. Since, however, the string scale ℓ_s is just the characteristic scale corresponding to the instability, we have to take into account tachyon, and all higher modes too which are characterized by the same string scale, in terms of open strings. In the case of pure D-particle systems, the validity of retaining only the lowest open string modes and consistency with supergravity at least in the lowest order approximation in the weak string coupling was ensured by the supersymmetry: It leads to the fact that both short-distance and long-distance forces are described by the lowest Yang-Mills modes alone in the approximation of one graviton exchange. Without manifest supersymmetry, however, there is no such mechanism which may ensure the validity of field-theory approximation.

A conclusion of this simple argument is that the D-particle and anti-D-particle system cannot be assumed to saturate the space-time uncertainty relation. In fact, if we just apply the ordinary Heisenberg uncertainty relation for the Hamiltonian $H = \frac{g_s \ell_s p^2}{2} - \frac{\ell_s^6}{r^7}$, we get a much larger spatial scale of order $\Delta X \sim g_s^{-1/5} \ell_s \gg \ell_s$. If we further assume that the scattering occurs through a metastable resonant state, the characteristic time scale is $\Delta T \sim g_s^{-7/5} \ell_s$ which leads to $\Delta X \Delta T \sim g_s^{-8/5} \ell_s^2 \gg \ell_s^2$. It seems that the saturation of the space-time uncertainty relation is expected only for some particularly symmetric systems, such as the systems satisfying the BPS condition and the generalized conformal symmetry. This expectation is also in accord with the result of high-energy fixed angle (or high-momentum transfer) scattering of strings, if one supposes that through the high-energy fixed angle scattering we are probing a regime where the symmetry is much enhanced.

At this juncture, it is perhaps worth remarking also that the generalized conformal symmetry is regarded as the underlying symmetry for the so-called DLCQ interpretation of the Yang-Mills matrix model. We can freely change the engineering scales in analyzing the system. Thus, if one wants to keep the numerical value of the transverse dimensions, we perform a rescaling $t \to \lambda^{-1} t, X_i \to \lambda^{-1} X_i, \ell_s \to \lambda^{-1} \ell_s$ simultaneously with the generalized scaling transformation leading to the scaling $t \to \lambda^{-2} t, X_i \to X_i, R \to \lambda^2 R$ and $\ell_{11} \to \ell_{11}$ which can be interpreted as the kinematical boost transformation along the 11th direction which is compactified with radius $R_{11} = g_s \ell_s$. Alternatively, we can keep the numerical value of time or energy by making a rescaling $t \to \lambda t, \quad X_i \to \lambda X_i, \ell_s \to \lambda \ell_s$, leading to the scaling $t \to t, R \to \lambda^4 R, X_i \to \lambda^2 X_i, \ell_{11} \to \lambda^2 \ell_{11}$ and $\ell_s \to \lambda \ell_s$, which is in fact equivalent to the 'tilde' transformation utilized in[48] in trying to justify the Matrix theory for finite N. Note that although the second case makes the string length ℓ_s small by assuming small λ, the length scale of transverse directions smaller than the string scale is always sent to even smaller length scale ($< \lambda^2 \ell_s$). For further discussions and applications of the generalized conformal symmetry in D-brane dynamics, we refer the reader to our papers cited above. Here we only mention that the generalized conformal symmetry provides a basis for the extension of the AdS/CFT correspondence for the Yang-Mills matrix model. The concrete computation of the correlators led to somewhat unexpected but suggestive results with respect to the question of the compatibility of Lorentz invariance and holography in Matrix-theory conjecture, as fully discussed in[45][46].

4.3. *Interpretation of black-hole complementarity and UV-IR correspondence*

In the first part of the present section, we have emphasized the relevance of the space-time uncertainty relation to the question of formation of microscopic black holes by the fluctuation of space-time geometry. Is it relevant also for macroscopic black holes? Qualitatively at least, one thing is clear. For the external observer sitting outside black holes, strings are seen to more and more spread as they approach to the horizon, because of the infinite time delay near the horizon. Namely, for the observer far from the horizon, the uncertainty of time becomes small $\Delta T \to 0$ without limit as strings approach to the horizon. The space-time uncertainty relation then demands that the spatial uncertainty increases as $\Delta X \sim \ell_s^2/\Delta T \to \infty$ without limit. This phenomenon is the basis for the proposal of implementing the principle of 'black-hole complementarity'[49] in terms of string theory by Susskind[50] in 1993. The general space-time uncertainty relation (2·2) proposed earlier just conforms to this principle of black-hole physics. In fact, a version of the space-time uncertainty relation has been rederived in light-cone string theory in[50] from the viewpoint of black-hole complementarity.

However, starting from the microscopic string theory, it is in general an extremely difficult dynamical question how to deal with macroscopic black holes, involving string interactions in essential ways and resulting in the macroscopic scales quite distant from the fundamental string scale. So we cannot be completely sure what are the concrete physical consequences of the above general property of strings near the horizon. In the present subsection, we give a reinterpretation of the Beckenstein-Hawking entropy of macroscopic black hole from the viewpoint of the space-time uncertainty relation following the general idea of black-hope complementarity. Although most of what we discuss here may be different ways of expressing the things which have been discussed previously, we hope that our presentation at least has a merit of looking important things from a new angle.

As already alluded to in our derivation of the space-time uncertainty relation, one of the crucial property of a free string, which is responsible for the space-time uncertainty principle, is its large degeneracy $d(E) \sim \exp k\ell_s E$ as energy increases. It is reasonable to suppose that this property is not qualitatively spoiled by the interaction of strings which must be definitely taken into account for the treatment of macroscopic phenomenon.

Based on this expectation, our fundamental assumption is that the entropy of macroscopic Schwarzschild black hole is given by

$$S = \log W \sim \Delta X_{eff}/\ell_s \tag{4·48}$$

where ΔX_{eff} is the *effective* spatial uncertainty of the state. The space-time uncertainty relation then leads to a lower bound in terms of the effective uncertainty ΔT_{eff} along time direction as,

$$S \gtrsim \ell_s/\Delta T_{eff}. \tag{4·49}$$

Intuitively, the motivation for this proposal should be clear : We have replaced the energy by the uncertainties in the formula of degeneracy $W \sim d(E)$ of a free string state. In particular, the form $\Delta X_{eff}/\ell_s$ is natural if we assume that the macroscopic state is effectively described as a single string state with effective longitudinal length ΔX_{eff} corresponding to the effective spatial uncertainty. The assumption that near a black hole horizon the state should be treated as a single string state seems natural in view of exponentially large degeneracy, as previously argued, *e.g.*, in[50].

The effective uncertainties in general should depend on how precisely the states are specified. The macroscopic state means that the state is specified solely by the macroscopic variables of state, such as the mass, temperature, total angular momentum, and so on. In the case of a Schwarzschild black hole, such macroscopic parameters are only its mass M and its Schwarzschild radius R_S. We treat these two parameters as being independent, since the gravitational constant is regarded as an independent dynamical parameter corresponding to the vacuum expectation value of dilaton. Now, on dimensional ground, the effective spatial uncertainty must take the form

$$\Delta X_{eff} = \ell_s f(\frac{R_S}{\ell_s}, M\ell_s). \tag{4·50}$$

However, the entropy of a macroscopic state should be expressible only in terms of the macroscopic parameters, the function $f(\frac{R_S}{\ell_s}, M\ell_s)$ actually depends only on the product of the variables.

$$f(\frac{R_S}{\ell_s}, M\ell_s) = f(R_s M).$$

To fix the form of the function $f(x)$ of a single variable, we here invoke the 'correspondence principle'[50)51)] that the black hole entropy must be reduced to $\log d(M) \sim \ell_s M$ at the point where the Schwarzschild radius becomes equal to the string scale $R_S \to \ell_s$, which immediately leads to $f(x) \sim x$, namely, the entropy of the Beckenstein form in D dimensional space-time,

$$S \sim R_S M \sim (G_D M)^{1/(D-3)} M \sim G_D^{-1} (G_D M)^{(D-2)/(D-3)} \tag{4·51}$$

where G_D is the Newton constant in D-dimensions, $G_D \sim g_s^2 \ell_s^{D-2}$.

The characteristic effective time uncertainty $\Delta T \sim \ell_s/R_S M$ associated with this reinterpretation of the black hole entropy can be understood from the viewpoint of 'stretched horizon' which is assumed to be located at a distance of order ℓ_s. As is well known, the near horizon geometry of a large Schwarzschild black hole is approximated by the Rindler metric $ds^2 = -\rho^2 d\tau_R^2 + d\rho^2 + ds^2_{transverse}$ whose time τ_R is related to the Schwarzschild time (namely time which is synchronized with a clock at infinity) by $\tau_R \sim t/R_S$. The time scale at the stretched horizon $\rho \sim \ell_s$ must be scaled by ℓ_s. Then, a Schwarzschild time scale of order $1/M$ is converted to a proper time scale $\ell_s/R_S M$ at the stretched horizon. Thus the *effective* uncertainties are essentially the uncertainties at the stretched horizon measured in the Rindler frame[50)] describing the near-horizon geometry of a macroscopic black hole.

Our arguments, though admittedly mostly the consequences of simple dimension counting and hence yet too crude, seems to show a basic conformity of the space-time uncertainty principle to black hole entropy, and perhaps to the property of holography [52)53)], which is expected to be satisfied in any well-defined quantum theory of gravity. The information of a macroscopic black hole is encoded within the spatial uncertainty of order $\Delta X_{eff} \sim R_S M \ell_s$. Or in terms of time, this corresponds to the effective time resolution of order $\Delta T_{eff} \sim \ell_s/R_S M$ at the lower bound for the entropy. At first sight, the last relation may look unwieldily counter intuitive, since it suggests a time scale much smaller than the string scale for understanding a macroscopic object. But it is not so surprising if we recall that this is precisely where the black-hole horizon plays the role as the agent for producing an infinite delay with respect to time duration. Although the horizon is not singular at all in terms of classical *local* geometry, it plays a quite singular role in terms of quantum theory, which *cannot* be formulated in terms of local geometry alone because of the superposition principle. This is one of the fundamental conflicts between general relativity and quantum theory, from a conceptual viewpoint. The space-time uncertainty relation demands that this conflict should be resolved by abandoning the simultaneous locality with respect to both time and space. In the previous section, we have seen that such a weakening of locality does not directly contradict the analyticity of S-matrix.

The proposed general form (4·48), in particular, its lower bound (4·49) suggests that to decode the information, it is in general necessary to make the time resolution large by appropriately averaging over the time scale, in accordance with a viewpoint expressed in[54)] in the context of Matrix theory. The time averaging in turn liberates the information stored in the spatial uncertainties and hence reduces the value of entropy.

For an observer outside black-hole horizon, decoding the whole information stored inside requires an observation of infinitely long time.

In connection with holography, we finally remark on the connection of the space-time uncertainty relation with the so-called UV-IR correspondence[56)], which is familiar in the recent literature of AdS/CFT correspondence[57)]. In brief, the UV-IR correspondence says that UV behaviors of the Yang-Mills theory (CFT) on the boundary corresponds to IR behaviors of supergravity in the bulk, and vice verse. On the other hand, the space-time uncertainty relation for open strings mediating D-branes leads to the similar relation that the small spatial uncertainty ΔX in the bulk corresponds to large uncertainties ΔT along the time-like direction on the brane at the boundary. Thus, they seem to be equivalent in the sense that UV and IR are correlated in the bulk and boundary. However, with a little scrutiny there is a small discrepancy in that the UV-IR relation is the statement involving classical supergravity and consequently that it only requires a macroscopic scale characterized by the curvature near the horizon, which is given as $R_{ads} \sim (g_s N)^{1/4}\ell_s$. In contrast with this, the space-time uncertainty relation only involves the string scale ℓ_s. This puzzle is resolved as suggested essentially in[55)] if we convert the uncertainty along the time-like direction into the spatial one on the brane at the boundary. Since, for the brane, open strings behave as electric sources, the uncertainty ΔT with respect to time are translated typically into the self energy associated with the spatial uncertainty ΔX_{brane} within the brane as

$$\Delta T \sim \Delta X_{brane}/\sqrt{g_s N}, \tag{4·52}$$

by using the well known fact that the effective Coulomb coupling for the superconformal Yang-Mills theory is $(g_s N)^{1/4} \sim (g_{YM}^2 N)^{1/4}$. This leads to $\Delta X_{bulk}\, \Delta X_{brane} \sim R_{ads}^2$ which is the one actually used in[56)55)], for a derivation of the holographic bound for the entropy of D3-branes. Note that here we are using the standard AdS coordinate used in[57)] instead of that of[56)]. The infrared cutoff of order $\Delta X_{bulk} \sim R_{ads}$ amounts to the ultraviolet cutoff of order $\Delta X_{brane} \sim R_{ads}$ for D-branes at the boundary. For D3 brane wrapping around a 3-torus of volume L^3, the degree of freedom is then $N_{dof} \sim N^2 L^3/R_c^3 = L^3 R_c^5/G_{10}$.

We emphasize that the holography and UV-IR correspondence are statements of macroscopic nature, involving only macroscopic parameters in their general expressions. In fact, black-hole entropy bound and more general holographic bound have been argued (see[58)] and references therein) to follow from the second law of thermodynamics, generalized to gravitating systems. In contrast to this, the space-time uncertainty relation is a general principle of microscopic nature, characterized directly by the string scale without any macroscopic variables. Hence, in applying the space-time uncertainty relation to macroscopic physics, it is in general necessary to make appropriate conversions of the scales in various ways depending on different physical situations, as exemplified typically by (4·48), (4·50) and (4·52). Qualitative conformity of the microscopic space-time uncertainty relation with holography suggests that the former can be a consistent microscopic principle which underlies correctly the required macroscopic properties. As emphasized above, the departure of string theory from the framework of local field theory seems to be minimal in its nature. But the nonlocality of string theory as being signified by the space-time space-time uncertainty principle appears to be sufficient for coping with black-hole complementarity and holography.

Finally, in connection with the problem of macroscopic black holes, there remains one big problem. That is the problem of space-time singularities. Customarily, we expect that the classical geometry breaks down around the length scale near the string scale ℓ_s. From the point of view of the space-time uncertainty relation, however, we have to discriminate the scales with respect to time and space. If we take the typical example of Schwarzschild black hole, the singularity is a space-like region. Any object after falling inside the horizon encounters the singularity within a finite proper time. If one asks precisely what time it encounters the singularity, the time resolution of the clock on the object must be sufficiently small. But then the space-time uncertainty relation again tells us that the locality with respect to the spatial direction is completely lost. Thus the classical local-geometric formulation which the existence of singularity relies upon loses its validity. Similarly, if the singularity is time like, the locality along the time direction is completely lost. It seems thus certain that in string theory space-time singularities are resolved. However, it is unclear whether this way of resolving the problem of space-time singularities has any observable significance, characterizing string theory.[*)]

§5. Toward noncommutative geometric formulation

We have emphasized the role of world-sheet conformal symmetry as the origin of the space-time uncertainty relation. As has already been alluded to in the end of subsection 2.1, such a dual relation between time and space obviously suggests some mathematical formalism which exhibits noncommutativity between operators associated to space and time. However, usual world-sheet quantum mechanics of strings does not, at least manifestly, show such noncommutativity. In a sense, in the ordinary world-sheet formulation, use is made of a representation in which the time (center-of-mass time of string) is diagonalized, and the spatial extension ΔX is measured by the Hamiltonian, as is evident in our first intuitive derivation of the space-time uncertainty relation. Thus the noncommutativity between space and time is indeed there in a hidden form. Are there any alternative formulations of string quantum mechanics which explicitly exhibit the noncommutativity? Note that we are not asking a further extension of string theory with an additional requirement of space-time noncommutativity. What is in mind here is a different representation of string theory with manifest noncommutativity, which is however equivalent, at the level of on-shell S matrix, to the usual formulation at least perturbatively. A different representation may well be more suited for off-shell non-perturbative formulation, hopefully.

The purpose of this section is to suggest a particular possibility along this direction. From the above consideration, we should expect the existence of a world-sheet picture which is quite different from the ordinary one with respect to the choice of gauge. Let

[*)] An interesting remark is that, in both cases of black-hole horizon and space-time singularity, the increase of spatial extendedness of strings in the short time limit is coincident with those of the spatial distances between the geodesic trajectories exhibited in the classical Schwarzschild metric.

us consider the so-called Schild action*) of the form ($\lambda = 4\pi\alpha', \xi = (\tau, \sigma)$)

$$S_{schild} = -\frac{1}{2}\int d^2\xi\, e\Big\{\frac{1}{e^2}[-\frac{1}{2\lambda^2}(\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu)^2] + 1\Big\} + \cdots \tag{5·53}$$

where e is an auxiliary field necessary to keep the reparametrization invariance. We will only consider the bosonic part for simplicity. The relevance of this action to the space-time uncertainty relation has already been discussed in a previous work[25] from a slightly different context. There, it was shown how to transform the action into the more familiar Polyakov formulation. Also it motivates the definition of a particular matrix model, called 'microcanonical matrix model', as a tentative nonperturbative formulation, by introducing a matrix representation of the commutation constraint (2·5). The latter model is quite akin to the type IIB matrix model[17].

From the point of view of conformal invariance, the equivalence of this action with the ordinary formulation is exhibited by the presence of the same Virasoro condition as the usual one. We can easily derive it as constraints in Hamiltonian formalism:

$$\mathcal{P}^2 + \frac{1}{4\pi\alpha'}\acute{X}^2 = 0, \quad \mathcal{P}\cdot\acute{X} = 0. \tag{5·54}$$

In deriving this relation, it is essential to use the condition coming from the variation of the auxiliary field e

$$\frac{1}{e}\sqrt{-\frac{1}{2}(\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu)^2} = \lambda. \tag{5·55}$$

which we proposed to call as 'conformal constraint' in[25]. Under this circumstance, we can proceed to the ordinary quantization with the Virasoro constraint as first class constraint. In this case, there is apparently no place where noncommutativity of space-time coordinates appears. The space-time uncertainty relation is embodied in conformal invariance which is typically represented by the Virasoro condition.

Now let us change to another possible representation of the Schild action by introducing a new auxiliary field $b_{\mu\nu}(\xi)$ which is a space-time antisymmetric tensor of second rank but is a world-sheet density,

$$S_b = -\frac{1}{2}\int d^2\xi\, e\Big\{\frac{1}{e^2}[\frac{1}{\lambda^2}(\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu b_{\mu\nu} + \frac{1}{2}b_{\mu\nu}^2)] + 1\Big\}. \tag{5·56}$$

This can further be rewritten by making a rescaling $b_{\mu\nu} \to e b_{\mu\nu}$ of the b field,

$$S_{b2} = -\int d^2\xi\,\Big\{\frac{1}{2\lambda^2}\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu b_{\mu\nu} + \frac{1}{2}e(\frac{1}{2\lambda^2}b_{\mu\nu}^2 + 1)\Big\}. \tag{5·57}$$

Note that the b field is then a world-sheet scalar. Usually, this Lagrangian is not convenient for quantization since it contains only first derivatives with respect to the world-sheet (proper) time, leading to second class constraints, and there is no kinetic term and no Hamiltonian. From the viewpoint of noncommutative space-time coordinates, on the other hand, the second class constraints making identifications between

*) The original action proposed in[59] did not contain the auxiliary field e. However, an equivalent condition was imposed by hand.

some components of momenta and coordinates, could be the origin of the noncommutativity. If for example we assume for the moment that the external b field is independent of the world-sheet time, the Dirac bracket taking account the second class constraint is

$$\{X^\mu(\sigma_1), X^\nu(\sigma_2)\}_D = \lambda^2((\partial_\sigma b(\sigma_1)^{-1})^{\mu\nu}\delta(\sigma_1-\sigma_2).$$

To see that this conforms to the space-time uncertainty relation, it is more appropriate to rewrite it as

$$\{X^\mu(\sigma_1), \frac{1}{\lambda}\partial_\sigma b^\mu_\nu(\sigma_2)X^\nu(\sigma_2)\}_D = \lambda\delta(\sigma_1-\sigma_2). \tag{5·58}$$

Since the b field satisfy the constraint equation, assuming that the auxiliary field e is first integrated over,

$$\frac{1}{2\lambda^2}b^2_{\mu\nu} = -1 \tag{5·59}$$

we must have nonvanishing time-like components b_{0i} of order λ

$$b^2_{0i} = \lambda^2 + \frac{1}{2}b^2_{ij} \geq \lambda^2.$$

Then (5·58) is characteristic of the noncommutativity between target time and the space-like extension of strings.

In general case of time dependent auxiliary field b, it is not straightforward to interpret the above action within the ordinary framework of canonical quantization, since the system is no more a conserved system, with explicit time dependence in the action. However, the essence of noncommutativity lies in the presence of phase factor itself

$$\exp\,[i\int d^2\xi\, \frac{1}{2\lambda^2}\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu b_{\mu\nu}],$$

rather than a formal interpretation in terms of operator algebra. The path integral in principle contains the whole information of both the operator algebra and its representation. Let us assume the appearance of this phase factor is an indispensable part of any quantization based on the action (5·57). Then, we can qualitatively see a characteristic noncommutativity between time and space directions directly in this phase factor for general case. To avoid a complication associated with the boundary we restrict ourselves to closed strings in the following discussions.

First, in the presence of this phase factor, the most dominant configuration for the b field for generic world-sheet configuration of the string coordinates are those with the smallest possible absolute values allowed under the constraint (5·59). For, the larger the absolute value of b is, the cancellation of the path integral over the world-sheet coordinate becomes stronger. So let us first consider the case where the spatial components are zero $b_{ij}=0$, leading $b^2_{0i}=\lambda^2$. The effect of spatial components b_{ij}, corresponding to the noncommutativity among spatial coordinates, will be briefly described later. Under this approximation, dependence on the world-sheet coordinate in the b field satisfying the constraint is expressed as a local O($D-1$) rotation belonging to a coset $O(D-1)/O(D-2)$,

$$b_{0i}(\tau,\sigma) = \lambda S_{ri}(\tau,\sigma). \tag{5·60}$$

Here we represent the coset element by the matrix elements S_{ri} with r being the radial direction for definiteness.

Let us now choose the time-like gauge

$$\partial_\sigma X^0 = 0$$

and treat the target time as a globally defined dynamical variable on the world-sheet as a function of the world-sheet time parameter τ. Then the phase factor reduces to

$$\exp\,[i\int d\tau \frac{1}{\lambda^2}\dot{X}^0 \int d\sigma b_{0i}(\xi)\partial_b X^i].$$

We can interpret this phase factor as arising from the product of short-time (with respect to world-sheet time) matrix element

$$\begin{aligned}&\langle X^0(\tau+\frac{1}{2}\epsilon)|X^0(\tau-\frac{1}{2}\epsilon)\rangle\\ &\quad=\int[d\vec{X}dS(\tau,\sigma)]\langle X^0(\tau+\frac{1}{2}\epsilon)|\vec{X},\partial_\sigma S\rangle\langle\vec{X},\partial_\sigma S|X^0(\tau-\frac{1}{2}\epsilon)\rangle\end{aligned} \tag{5·61}$$

where the intermediate state to be integrated over is inserted at the mid-point and the matrix elements are

$$\langle X^0(\tau+\frac{1}{2}\epsilon)|\vec{X},\partial_\sigma S\rangle = \exp\,[i\frac{1}{\lambda^2}X^0(\tau+\frac{1}{2}\epsilon)\int d\sigma b_{0i}(\xi)\partial_b X^i] \tag{5·62}$$

$$\langle\vec{X},\partial_\sigma S|X^0(\tau-\frac{1}{2}\epsilon)\rangle = \exp\,[-i\frac{1}{\lambda^2}X^0(\tau-\frac{1}{2}\epsilon)\int d\sigma b_{0i}(\xi)\partial_b X^i]. \tag{5·63}$$

In a more familiar operator form, this would correspond to a commutator

$$[X^0,\int d\sigma\, S_{ri}\partial_\sigma X^i] = i\lambda$$

at each instant of world-sheet time. But the phase factors as exhibited in (5·62) or (5·63) directly lead to an uncertainty relation of the following form, just by the same mechanism as the ordinary Fourier transformation,

$$|\Delta X^0||\Delta\vec{X}| \gtrsim \lambda \tag{5·64}$$

$$|\Delta\vec{X}| = \sqrt{\langle\Big(\Delta\int d\sigma S_{ri}\partial_\sigma X^i(\sigma)\Big)^2\rangle} \tag{5·65}$$

with respect to the orders of magnitude for uncertainties in the path integral. We note that (5·65) is invariant under reparametrization with respect to σ. Furthermore, the latter is acceptable as a measure for the spatial uncertainty, since it locally measures the length along the tangent of the profile of closed strings at fixed world-sheet time including the possibility of multiple winding, provided it does not vanish. In particular, when $X^i(\sigma)$ and $S_{ri}(\sigma)$ as two vectors are parallel to each other along the string, it precisely agrees with the proper length measured along the string. For general random configurations of the orientation of these vectors, (5·65) is a possible general definition of the length of a string in a coarse-grained form.

The effect of spatial components b_{ij} can be taken into account if we generalize the local rotation to local Lorentz group $O(D-1,1)$ in (5·60). This is due to the fact that we can restrict the component of the auxiliary field b to those which have nonvanishing product $b_{\mu\nu}\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu$. Since the antisymmetric tensor $\sigma_{\mu\nu} = \epsilon^{ab}\partial_a X^\mu \partial_b X^\nu/2$ can locally be transformed to that corresponding to a time-like two-dimensional plane,*) we can assume a parametrization, say $b_{\mu\nu} = \lambda S_{0\mu}S_{r\nu}$ using the rotation matrix of $O(D-1,1)$. This leads to a correction to the definition of the spatial uncertainty as

$$|\Delta\vec{X}| = \sqrt{\langle\Big(\Delta\int d\sigma(S_{00}S_{ri} - S_{0i}S_{r0})\partial_\sigma X^i(\sigma)\Big)^2\rangle}.$$

Also, there arises an induced noncommutativity among the spatial components, corresponding to the following phase factor

$$\exp\,[i\frac{1}{\lambda^2}\int d\tau d\sigma\, \dot{X}^i X'^j(S_{0i}S_{rj} - S_{0j}S_{ri})].$$

This should be interpreted as the residual noncommutativity which is necessary to preserve Lorentz invariance in the presence of the primary noncommutativity between time and space.

Although a more rigorous formulation is desirable, our discussion seems to already suggest a quite remarkable possibility that the space-time noncommutativity alone governs the essential features of the dynamics. This would not be so surprising if we remember that the space-time uncertainty relation can be regarded as a reinterpretation of the time-energy uncertainty relation. As such, its proper formulation would necessarily amount to formulating the Hamiltonian appropriately, as should have been clear from our foregoing discussions.

Of course, this particular formalism does not seem convenient for performing concrete computations of string amplitudes, at least by technical tools presently available for us. Also, our discussion, being based upon the world-sheet picture, is yet perturbative in its nature. As we have stressed, the space-time uncertainty relation should be valid nonperturbatively, and hence must be ultimately reformulated without relying upon the world-sheet picture on the basis of some framework which is second-quantized from the outset. The connection with matrix models discussed in a previous work[25] is certainly suggestive toward a nonperturbative formulation, but unfortunately seems lacking yet some key ingredients for a definitive formulation. We hope, however, that the above argument gives some impetus for further investigations toward really nonperturbative and calculable formulations in the future. For example, from the viewpoint of an analogy between classical phase space and space-time that we have mentioned in discussing the generalized conformal transformation, the study of the most general transformations which leave the form $i\int d^2\xi\,\frac{1}{2\lambda^2}\epsilon^{ab}\partial_a X^\mu\partial_b X^\nu b_{\mu\nu}$ might be a direction to be pursued.

In connection with this, it might be possible to reinterpret directly the action (5·57) as a generalized deformation quantization of space-time geometry itself. This expectation also suggests a formulation from the viewpoint of M-theory by interpreting

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*) In terms of invariants, this corresponds to the following property. Let $\sigma^2_{\mu\nu} = \Sigma_{\mu\nu}$. Then $\Sigma^2_{\mu\nu} = \mathrm{Tr}(\Sigma)\Sigma_{\mu\nu}/2$. Thus there is only one independent Lorentz invariant.

the world sheet of strings as a section of membrane, and using a sort of the formalism related to the Nambu bracket[60]. We also mention that to make the comparison with local field theory, the approach suggested in[61] might be of some relevance in the case of open strings. We left all these possibilities as challenging and promising problems for the future.

Most of the readers must have noticed a similarity of the appearance of noncommutativity with that of the recent discussions of noncommutative Yang-Mills theory based on D-branes. An obvious difference is that our b field is a world-sheet field which is always there without the presence of the external space-time B field. Note that we obtained the noncommutativity in the sense of target space-time from the world sheet b field in the bulk of string world sheet. But this noncommutativity is nothing but another representation of the space-time uncertainty relation already exhibited in the usual formulation with manifest conformal symmetry. Also, in our case, the dominant components of the b field is the time-like components b_{0i}, contrary to the space-like components of B field in[62][63]. If we had treated D-branes using the above formulation based on the Schild action for open strings attached to D-branes by adding the constant space-time B_{ij} field, we would get the noncommutativiy between time and space directions as above along the D-brane world volume, in addition to the noncommutativity among spatial directions along D-branes in association with B_{ij}.

Our approach to noncommutativity is also quite different from that of[64] in type IIB matrix models. However, since the Schild action is intimately connected to the type IIB matrix model, it would be very interesting to seek some possible relation with it.

We emphasize again that the noncommutativity exhibited in the present section between time and space is a property which is intrinsic to the dynamics of fundamental strings, and is nothing to do with the presence or absence of the external B field. Of course, the space-time B field is automatically contained as a state of closed strings in any valid formulation of string theory. In quantum theory, we have to take into account of its vacuum fluctuation. In this broad sense, these two different origins of space-time noncommutativity might be united in some nonperturbative framework, by identifying the fluctuation of the space-time B field and the world sheet b field self-consistently.

§6. Further remarks

In this final section, we discuss some miscellaneous questions which have not been treated in the preceding sections and may sometimes become the sources of confusion. We also comment on some future possibilities.

Frame dependence, (p, q) strings, and S-duality

Since the space-time uncertainty relation is a statement which contains a dimensionful parameter ℓ_s, we have to specify the frame for the metric in the sense of Weyl transformation, with respect to which the string length parameter is defined. In the foregoing discussions, we were always tacitly assuming that the string length ℓ_s is the proportional constant in front of the world-sheet string action, say $(1/\ell_s^2)\int d^2\xi g_{\mu\nu}\partial_z X^\mu \partial_{\bar{z}} X^\nu + \cdots$ using the standard conformal gauge. Therefore the frame of the space-time metric $g_{\mu\nu}(X)$ which should be used for the space-time uncertainty relation is the so-called string frame metric. This is important when we consider the S-duality transformation, under which the string metric is not invariant.

Suppose we start with the fundamental string ($(1,0)$ string) in type IIB theory and make a S-duality transformation which send $(1,0)$ strings to (p,q) strings. In the original (1,0) picture, the other (p,q) strings are soliton excitations. Therefore, their interaction and motion are governed by the fundamental strings. In this sense, the space-time uncertainty relation must be satisfied using the original string frame metric at least in a weak coupling regime where the tension of (1,0) string is smaller than (p, q strings, provided we correctly identify the uncertainties. Note that the same can be said for other higher dimensional D-branes. *) As long as we consider them in the weak string coupling regime with respect to the original fundamental string, whole of the dynamics is basically expressible in terms of the fundamental strings. Although we now know that string theory is full of objects of various dimensions, they cannot be treated in a completely democratic way from the point of view of real dynamics of them.

However, if we want to use the picture in which the (p,q) string is now treated as fundamental one in the regime where the transformed string coupling $g_s^{(p,q)} = \exp\phi_{(p,q)}$ is weak and hence the original string coupling is in general in a strong-coupling regime, we have to use the world-sheet action of the (p,q) string to describe the dynamics. Then it is essential to shift our frame correspondingly. Namely, the space-time string metric must also be transformed by the same S-duality transformation. This precisely cancels the difference of tensions between $(1,0)$ and (p,q). This is of course as it should be as far as the S-duality transformation is a *symmetry* of the type IIB superstring theory. The space-time uncertainty relation is therefore invariant under the S-duality transformation. Thus at least in S-duality symmetric theories, the space-time uncertainty relation must be valid for arbitrary string coupling, provided the appropriate change of Weyl frame is made according to the transformation law of S-duality and the uncertainties are redefined correspondingly.

In formulas, it goes as follows. The world-sheet bosonic action for the (p,q) string is, using the ordinary string metric of target space-time as fundamental $(1,0)$ string,

$$T_{(p,q)}\int d^2\xi\, g_{\mu\nu}(X)\partial_z X^\mu \partial_{\bar z} X^\nu$$

where the tension of the (p,q) string in the original string frame unit is given by[66)]

$$T_{(p,q)} = \bar{\triangle}^{1/2}_{(p,q)}\frac{1}{\ell_s^2} \tag{6·66}$$

and

$$\bar{\triangle}_{(p,q)} = |p-q\rho|^2 = \exp(\phi_{(p,q)} - \phi_{(1,0)}) \tag{6·67}$$

with $\rho = \frac{\chi}{2\pi} + ie^{-\phi}$. On the other hand, the space-time string metrics are related by

$$g^{(p,q)}_{\mu\nu}(X)\exp(-\phi_{(p,q)}/2) = g^{(1,0)}_{\mu\nu}(X)\exp(-\phi_{(1,0)}/2)$$

*) For a discussion of some uncertainty relations along the D-brane world volume, see[65)].

with $g_{\mu\nu} = g_{\mu\nu}^{(1,0)}$, corresponding to the S-duality invariance of the Einstein frame metric. Combining these relations, we confirm that the world-sheet action of the (p,q) string is equal to

$$\frac{1}{\ell_s^2}\int d^2\xi\, g_{\mu\nu}^{(p,q)}(X)\partial_z X^\mu \partial_{\bar{z}} X^\nu .$$

Thus we have the space-time uncertainty relation with the same string length ℓ_s as that before making the transformation.

Curved or compactified space-time, and a remark on T-duality

Another remark related to the above question is that the space-time uncertainty relation must be valid qualitatively in general curved space-times allowed as backgrounds of string theory, as far as the world-sheet conformal invariance is not violated. In this case too, it is essential to use the string frame metric to measure the invariant (or proper) length appropriately with respect to time and spatial directions.*)

A somewhat related, but different question is the interpretation of T-duality from the viewpoint of space-time uncertainty relation. T-duality says that under the compactification of a spatial direction along a circle, the theory with a radius R is equivalent with that with ℓ_s^2/R. This is due to the mapping, $n \to m, R \to \ell_s^2/R$, between the momentum modes whose mass spectrum is n/R and the winding modes whose spectrum is mR/ℓ_s^2. From the viewpoint of space-time uncertainty relation, the uncertainty with respect to the former, referring only to the center-of-mass momentum, must be translated into an uncertainty with respect to energy by

$$\Delta T_1 \sim R_1/\Delta n_1$$

which imply the lower bound for the spatial uncertainty $\Delta X_1 \sim \ell_s^2 \Delta n_1/R_1$. Here we put the label 1 to denote the uncertainty relation in theory 1. Suppose the theory 1 is mapped into a theory 2 which is compactified with a radius R_2, by identifying the spatial uncertainty $\Delta X_1 \to \Delta X_2 = R_2 \ell_s \Delta m_2$ originated from the uncertainty with respect to the winding number, giving $\Delta T_2 \sim \ell_s^2/R_2 \Delta m_2$. Thus the uncertainty relations of both theories are related to each other by making the mapping

$$n_1 \to m_2, \quad m_1 \to n_2, \quad R_1 \to \ell_s^2/R_2.$$

This is precisely the mapping of T-duality transformation. Thus T-duality is consistent with the space-time uncertainty relation, as it should be. In connection with this, it must be kept in mind that for the uncertainty with respect to spatial directions, we have to take into account windings. For example, the definition of the spatial uncertainty suggested from the Schild action as discussed in the last section indeed naturally contains the winding effect. Another remark is that in our interpretation T-duality is a statement about duality between short and large distances in time and spatial directions, rather than on the existence of minimal distance as often expressed in the literacture.

The role of supersymmetry ?

*) For example, the discrepancy claimed in ref.[67] can easily be corrected by using the proper length appropriately.

In our discussions, the space-time supersymmetry has not played fundamental roles. The reason is that the supersymmetry is not directly responsible for the short distance structure of string theory. It rather plays a central role in ensuring the theory be well defined, at least perturbatively, at long distance regime. However, the space-time uncertainty relation essentially demands dual roles between ultraviolet and infrared regimes by interchanging the temporal and spatial directions. In this sense, the space-time supersymmetry must actually be playing an important subsidiary role in order to make the theories well-defined in both ultraviolet and infrared regimes. Such an instance was already explained for the case of D-particle dynamics.

In connection with this, a question arises whether we have to impose, in future non-perturbative formulations of string theory, supersymmetry as an additional assumption which is not automatically guaranteed from the fundamental principles alone. Although we do not know the answer, recent developments[68)] on unstable D-brane systems indicates that the mere appearance of tachyon should no more be regarded as the criterion of unacceptable theories. That only signifies a perturbative vacuum we have to start with is wrongly chosen. Indeed, it was recently shown by the present author[69)] that the 10 dimensional (orientable) open string theory with both bosons and fermions, either its Neveu-Schwarz-Ramond or Green-Schwarz formulation, have a hidden N=2 space-time supersymmetry automatically without making the standard GSO projection. It is an important question whether similar interpretation is possible for closed string theories as well.

M-theory interpretation of the space-time uncertainty relation ?

Let us next reconsider what is the relevance of the space-time uncertainty principle to the M-theory conjecture. In section 4, we have derived the M-theory scales from two different points of view, namely microscopic black hole in 10 dimensional space-times and D-particle dynamics. In particular, the former argument shows that the appearance of the M-theory scale can be a quite general phenomenon, not necessarily associated with D-branes.

One of the basic elements of the M-theory conjecture is that in 11 dimensions the role of fundamental strings is replaced by membranes, which is wrapped about the compactified circle of radius $R_{11} = g_s \ell_s$. From this point of view, it seems natural[38)71)] to further reinterpret the space-time uncertainty relation as

$$\Delta T \Delta X \gtrsim \ell_s^2 \sim \ell_M^3 / R_{11} \ \rightarrow \Delta T \Delta \vec{X} \Delta X_{11} \gtrsim \ell_M^3 \sim G_{11} \tag{6·68}$$

by setting $\Delta X_{11} \sim R_{11}$ as the uncertainty along the 11 the direction and $\Delta X \rightarrow \Delta \vec{X}$ which is identified to be the spatial uncertainty in the 9 dimensional transverse directions. This is in accord with the membrane action with the two space-like directions are along the world volume of membrane. In reference[38)], we have discussed affinity of this relation with AdS/CFT correspondence in 11 dimensions. This also motivated a study of the Nambu bracket in[70)]. The original stringy space-time uncertainty relation would then be an approximation to this relation in the limit of small compactification radius. Once we move to this viewpoint, the fundamental scale is now $\ell_M = \ell_{11} \sim g_s^{1/3} \ell_s$. Of course, any genuinely 11 dimensional effects only appear for large compactification radius $R_{11} \gg \ell_M$. In this regime, all the characteristic scales of the theory are governed by the order ℓ_M. The appearance of different scales for time and spatial scales in 10 dimensions controlled by the string coupling is obviously the effect of the small compactification scale R_{11}.

For example, we can apply the same argument for microscopic black hole as the criterion where truly M-theory effects take place. The black hole uncertainty relation places a restriction in 11 dimensions as

$$\Delta T(\Delta X)^8 \gtrsim \ell_M^9. \tag{6·69}$$

Comparing with the M-theory uncertainty relation (6·68), we find that the critical point is of the same order

$$\Delta T_c \propto \Delta X_c \propto \ell_M,$$

if we treat all the spatial directions equivalently, as is evident from the outset since there can be no other scales than ℓ_M unless one puts them in by hands. Therefore in this case, the dimensionless proportional coefficients are very important in order to ascertain various characteristic scales. In this sense, in M-theory, understanding of the real nonperturbative mechanisms for generating the low-energy scales becomes completely nonperturbative, at a much higher level than in 10 dimensional string theory.

We also note that it is straightforward to extend the Schild action approach introduced in section 5 toward a noncommutative geometric formulation for the quantization of membrane. In this case, the role of the world-sheet auxiliary field $b_{\mu\nu}$ is played by a world-volume 3 rank tensor field $c_{\alpha\beta\gamma}(\xi)$. We can easily derive an analog of the stringy uncertainty (5·65) for membrane.

Quite recently, it has been argued[72] that the relation (6·68) is compatible with the so-called 'stringy exclusion principle'[73] on AdS space-times, by reinterpreting an observation made in[74]. Also an approach proposed in[75] to the stringy exclusion principle suggests a connection with the quantum group interpretation, another possible manifestation of noncommutativity, of these phenomena.

A fundamental question

In the beginning of this paper, we have repeatedly stressed the importance of reinterpreting the role of world-sheet conformal symmetry in terms of some new language, which is not in principle dependent upon perturbation theory, as a motivation of our proposal of the space-time uncertainty principle. There however remains still one of the most mysterious questions in string theory. Why does string theory contain gravity?[*] Of course, we have checked the consistency of the space-time uncertainty relation with the presence of gravity from various viewpoints. In spite of many such checks, it is still unclear, unfortunately, what ensures the appearance of general relativity at long distance regime. The main reason for this deficiency is that we have not gained appropriate understanding on the symmetries associated with the space-time uncertainty principle in terms of the target space-time. The generalized conformal symmetry we have mentioned in section 4 might contain some ideas which might form a germ for investigation toward such directions.

Although a lot of questions still remain, summarizing all what we have discussed in the present paper, it seems not unreasonable to assert that the space-time uncertainty

[*] For a recent general review on this question, see[76].

principle may be one of possible general underlying principles governing the main qualitative features of string/M theory. Of course, the scope of qualitative principles, such as our space-time uncertainty principle, is very limited to make any concrete predictions without having definite mathematical formulations. In this paper, we have tried to clarify its meaning and implications as far as we can at the present stage of development. It would be extremely interesting to arrange various aspects discussed here into a unified mathematical scheme.

Acknowledgements

The present paper was essentially completed during the author's visit at Brown University in March, 2000. He would like to thank Department of Physics, Brown University for the hospitality and also discussions with D. Lowe, A. Jevicki and S. Ramgoolam during preparation of the manuscript. The present work is supported in part by Grant-in-Aid for Scientific Research (No. 09640337) and Grant-in-Aid for International Scientific Research (Joint Research, No. 10044061) from the Ministry of Education, Science and Culture.

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